# Sraffa's incomplete reductions to labor

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## Abstract

Sraffa, in part 1 of his 'Production of Commodities by Means of Commodities', describes two kinds of reductions to labor. First, he reduces Classical natural prices to an infinite series of 'dated' quantities of labor multiplied by a profit factor. He concludes that prices are 'in proportion to their labor cost', that is Classical labor-values, only in the special case of zero profit. I show that Sraffa's reduction is incomplete in the precise sense that it ignores some actual labor supplied during the 'successive stages of the production of the commodity'. The complete reduction to dated quantities reveals that natural prices are, in fact, proportional to total labor costs. Second, Sraffa constructs a 'standard commodity' that functions as an 'invariable standard of value' in the context of changes in income distribution. Sraffa reduces the standard commodity, which he views as 'a purely auxiliary construction', to the 'variable quantity of labor' it commands in the market. I show that Sraffa's reduction is incomplete because it does not reduce the 'invariable standard' to a real cost of production. The complete reduction reveals that Sraffa's 'variable quantity' is, in fact, the total labor cost of the standard commodity. I conclude by discussing how Sraffa's incomplete reductions derive from the Classical category-mistake of conflating technical with total labor costs.

Keywords: Sraffa, standard commodity, classical economics, labor theory of value

JEL: B51, E11, D46

DRAFT JUNE 2013: PLEASE DO NOT CIRCULATE

## 1. Natural prices

Sraffa (1960), in Part 1 of 'Production of Commodities by Means of Commodities' (PCMC), specifies natural, or production, prices in terms of sets of simultaneous equations. Sraffa's work therefore forms part of the tradition of linear production theory (Gale, 1960; Pasinetti, 1977; Kurz and Salvadori, 1995), which includes notable precursors such as Quesnay's *Tableau Economique* (1758), Marx's reproduction schemes (Marx, [1893] 1974), and the work of Dmitriev (1974) and Leontief (1991).

In Chapter 1, 'production for subsistence', Sraffa examines a multisector economic model, formally similar to a closed Leontief model, that 'produces just enough to maintain

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[1] This work is the result of my current PhD studies supervised by Andrew Trigg at the Open University. Thanks to Robert Vienneau and Fernando Martins for comments on an earlier draft.

itself' including the 'necessaries for the workers' (Sraffa, 1960, p. 3). Sraffa notes the existence of a unique set of relative prices, defined by the technique, that 'if adopted by the market' would make it 'possible for the process to be repeated' (Sraffa, 1960, p. 3). At these natural prices the outputs of each sector can be exchanged to restore the original input distributions and, in consequence, the economy may reproduce at the same scale and in the same proportions.

Sraffa's 'subsistence' prices are equivalent to equation $\mathbf{pA} = \mathbf{p}$, which states that every commodity's cost of production, $\mathbf{pA}$, equals its selling price, $\mathbf{p}$, where $\mathbf{A}$ is a $n \times n$ input-output matrix. The prices constitute $n$ unknown variables. Assume matrix $\mathbf{A}$ has a dominant eigenvalue of 1, which implies the economy can produce exactly what it consumes. Assume matrix $\mathbf{A}$ is of full rank and irreducible. Then we can solve the equation to yield $n-1$ relative prices. Since one degree-of-freedom remains undetermined the solution is a price ray.

In Chapter 2, 'production with a surplus', Sraffa considers an economy that 'produces more than the minimum necessary for replacement and there is a surplus to be distributed' (Sraffa, 1960, p. 6). The surplus is an excess output or net product, which can be distributed either as additional consumption for workers or capitalists, or additional investment for capital accumulation and economic growth.

Sraffa, in essence, assumes matrix $\mathbf{A}$ has a dominant eigenvalue less than 1. The economy is then able to produce more than it consumes: more 'comes out' than 'goes in'. Given a 'physical' output that exceeds the used-up 'physical' inputs, and constant prices for the period under consideration, then necessarily output prices exceed input costs. So the original 'subsistence' price equation now becomes the *inequality*, $\mathbf{pA} < \mathbf{p}$, which states that every commodity's cost of production is less than its selling price. Profit is now possible. The existence of a surplus breaks the symmetry of the original price equation and prices are now under-determined. Sraffa remarks that 'the system becomes self-contradictory' (Sraffa, 1960, p. 6) because the left and right-hand sides of the equation no longer balance.

The production of a surplus raises the 'difficulty' of specifying relative prices that make it 'possible for the process to be repeated'. Sraffa adopts the Classical point-of-view that repeatability requires a uniform profit-rate otherwise capitalists would reallocate their capital and thereby alter the relative quantities produced in each sector.

Sraffa introduces the distributional variables, the profit-rate, $r$, and wage-rate, $w$, to construct a new price equation,

$$\mathbf{pA}(1+r) + \mathbf{l}w = \mathbf{p}, \tag{1}$$

which restores the equality between the left and right-hand sides, where $\mathbf{l}$ is a vector of direct labor coefficients. Natural prices comprise (i) the cost of means of production, $\mathbf{pA}$, (ii) the profit on the money-capital advanced, $\mathbf{pA}r$, and (iii) the cost of labor, $\mathbf{l}w$. Prices $\mathbf{p}$ are positive if $0 \leq r \leq R = (1/\lambda) - 1$, where $\lambda$ is the dominant eigenvalue of $\mathbf{A}$ and $R$ is the maximum profit-rate of the economic system; see Pasinetti (1977, pp. 95–97).

Sraffa explores the space of possible natural prices, by conjecturally varying $w$ and $r$, which fix different shares of the physical surplus that could be purchased by workers and capitalists. However, as Ravagnani (2001) notes, 'Sraffa never introduces in his analysis any specific assumption about the allocation of the physical surplus', i.e. Sraffa does not specify the actual commodity bundles distributed to the population.[2]

## 2. The 'reduction to dated quantities of labor'

Sraffa rewrites his natural price equation (1) in the equivalent form of an infinite series[3], or 'reduction equation' (Sraffa, 1960, p. 35):

$$\begin{aligned} \mathbf{p} &= \mathbf{l}w + \mathbf{lA}w(1+r) + \mathbf{lA}^2w(1+r)^2 + \cdots + \mathbf{lA}^n w(1+r)^n + \dots \\ &= \sum_{n=0}^{\infty} \mathbf{lA}^n w(1+r)^n, \end{aligned}$$

which he hypothetically interprets as a 'sum of a series of terms when we trace back the successive stages of the production of the commodity' (Sraffa, 1960, p. 89). The reduction reveals how prices resolve into functional income categories, that is payments to workers and capitalists. The $n$th term is the production costs, in terms of wages and profit, incurred $n$ 'years' prior to final output.

[2]Ravagnani therefore argues that Sraffa's approach is not restricted to self-reproducing states but has more general applicability. All statements in this paper are restricted to self-reproducing states with given gross outputs and therefore independent of this interpretative issue and any assumption regarding returns to scale. For an approach to value theory in the context of out-of-equilibrium dynamics, including scarcity prices, see Wright (2011b).

[3]Simply rearrange the price equation and note that $(\mathbf{I} - \mathbf{A}(1+r))^{-1} = \sum_{n=0}^{\infty} \mathbf{A}^n(1+r)^n$ for $0 \leq r \leq R$.

For example, in year $n = 0$, imagine capitalists sell unit outputs and pay workers $\mathbf{l}w$ in wages. In the previous year, $n = 1$, capitalists advanced $\mathbf{lA}w$ in wages to pay the labor that transforms means of production, $\mathbf{A}$, into unit outputs for sale the following year. The advanced wages are therefore 'tied up' in production for 1 'year'. The total costs incurred in year $n = 1$, then, are wages plus 1 'year' of profit on the advance, i.e. $\mathbf{lA}w(1 + r)$. In general, wages advanced in year $n$ do not return to the capitalist until $n$ years later when outputs are sold. Investments of different duration earn an equal return, or uniform profit-rate, by the application of compound interest. In consequence labor costs are 'multiplied by a profit factor at a compound rate for the appropriate period' (Sraffa, 1960, p. 34). Sraffa's reduction is therefore a series of terms that specify the wages of 'dated quantities of labor' (Sraffa, 1960, p. 34) plus profit compounded over the duration of investment, i.e. $\mathbf{lA}^n w(1 + r)^n$.

Classical labor-values, $\mathbf{v} = \mathbf{vA} + \mathbf{l}$, measure the direct ($\mathbf{l}$) and indirect ($\mathbf{vA}$) labor required to reproduce unit commodities. If capitalist profits are zero, i.e. $r = 0$, then the reduction equation yields $\mathbf{p} = \sum \mathbf{lA}^n w$ and natural prices are a simple sum of wage costs. Prices are therefore proportional to labor-values, i.e. $\mathbf{p} = \mathbf{v}w$. Sraffa states, therefore, that when the surplus is entirely distributed as wages 'the relative values [prices] of commodities are in proportion to their labor cost, that is to say to the quantity of labor which directly and indirectly has gone to produce them. *At no other wage-level do values* [prices] *follow a simple rule*' (Sraffa, 1960, p. 12) (my emphasis). In capitalist conditions, where profit is non-zero, natural prices do not simply vary with labor costs but also vary with the profit-rate. In consequence, natural prices are not proportional to Classical labor-values, except in special cases (Wright, 2009, 2011a).

Natural prices are an amalgam of labor costs and compound profits. Ricardo ([1817] 1996) therefore suggested that profit is 'only a just compensation for the time that profits were withheld'. Natural prices, it appears, are partially determined by a period of 'waiting' entirely unrelated to labor costs.

## 3. The complete reduction to dated quantities of labor

Consider 'production with a surplus' from the point of view of quantities $\mathbf{q}$ rather than prices $\mathbf{p}$. Quantities satisfy the inequality, $\mathbf{qA}^{\mathrm{T}} < \mathbf{q}$, which states that, for each commodity, the quantity used-up as inputs is less than the quantity output. A physical surplus is now possible dual to profits in the price system. To restore equality we must specify the distribution of real income.

Assume that the surplus, or net product, is distributed in the form of real wage, $\mathbf{w} = [w_i]$, and capitalist consumption bundle, $\mathbf{c} = [c_i]$. The quantity equation is then

$$\mathbf{q}\mathbf{A}^{\mathrm{T}} + \mathbf{w} + \mathbf{c} = \mathbf{q}, \tag{2}$$

which describes a self-reproducing state where the physical surplus is consumed by workers and capitalists.

In a self-reproducing state, the distribution of nominal income, specified by the profit and wage-rate, $w$ and $r$, is sufficient to purchase the real income, specified by $\mathbf{w}$ and $\mathbf{c}$. The distribution of real and nominal income are necessarily linked. Indeed, price equation (1) and quantity equation (2) imply

$$\mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}} r + \mathbf{l}\mathbf{q}^{\mathrm{T}} w = \mathbf{p}\mathbf{w}^{\mathrm{T}} + \mathbf{p}\mathbf{c}^{\mathrm{T}}, \tag{3}$$

which states that total profit, $\mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}} r$, and total wage income, $\mathbf{l}\mathbf{q}^{\mathrm{T}} w$, equals the cost of the net product, $\mathbf{p}\mathbf{w}^{\mathrm{T}} + \mathbf{p}\mathbf{c}^{\mathrm{T}}$. Assume further that workers and capitalists spend what they earn; in consequence,

$$\mathbf{l}\mathbf{q}^{\mathrm{T}} w = \mathbf{p}\mathbf{w}^{\mathrm{T}} \tag{4}$$

and

$$\mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}} r = \mathbf{p}\mathbf{c}^{\mathrm{T}}. \tag{5}$$

Equation (4) states that wage income equals the price of the real wage and equation (5) states that profit income equals the price of capitalist consumption. Together equations (4) and (5) link the distribution of real and nominal income.

Once we consider the distribution of income, in both nominal and real terms, important conclusions follow. Substitute $r = \mathbf{p}\mathbf{c}^{\mathrm{T}}/\mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}}$ (from equation (5)) into Sraffa's price equation (1):

$$\begin{aligned}
\mathbf{p} &= \mathbf{p}\mathbf{A}(1 + \frac{\mathbf{p}\mathbf{c}^{\mathrm{T}}}{\mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}}}) + \mathbf{l}w \\
&= \mathbf{p}(\mathbf{A} + \frac{1}{\mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}}}\mathbf{c}^{\mathrm{T}}\mathbf{p}\mathbf{A}) + \mathbf{l}w \\
&= \mathbf{p}\mathbf{A} + \mathbf{p}\mathbf{C} + \mathbf{l}w,
\end{aligned}$$

where matrix $\mathbf{C} = [c_{i,j}]$, such that

$$c_{i,j} = \frac{\mathbf{pA}^{(j)}}{\mathbf{pAq}^{\mathrm{T}}} c_i, \tag{6}$$

and $\mathbf{A}^{(j)}$ denotes the $j$th column of matrix $\mathbf{A}$.

What is matrix $\mathbf{C}$ in this equation? The meaning of each element $c_{i,j}$ becomes clearer if we multiply the numerator and denominator by the profit-rate,

$$c_{i,j} = \mathbf{pA}^{(j)} r \frac{c_i}{\mathbf{pAq}^{\mathrm{T}} r}.$$

The term $\mathbf{pA}^{(j)} r$ is the profit income generated by the sale of one unit of commodity $j$. The fraction $c_i / \mathbf{pAq}^{\mathrm{T}} r$ is the quantity of commodity $i$ consumed by capitalists per unit of profit income. Each element $c_{i,j}$ is therefore the quantity of commodity $i$ distributed to capitalists per unit output of commodity $j$. Matrix $\mathbf{C}$, in consequence, is a 'capitalist consumption matrix' that specifies how the production of output is synchronized with the distribution of goods from firms to capitalist households. Note that matrix $\mathbf{C}$ is a 'physical' input-output matrix that specifies relative material flows of commodities; for example, each element $c_{i,j}$ of $\mathbf{C}$ is a quantity measured in units identical to the corresponding element $a_{i,j}$ of the technique $\mathbf{A}$.

Sraffa's price equation (1) therefore has the equivalent form

$$\mathbf{pA} + \mathbf{pC} + \mathbf{l}w = \mathbf{p}, \tag{7}$$

where the real distributional variable $\mathbf{C}$ has replaced the nominal distributional variable $r$. Equation (7) provides an alternative, but quantitatively equivalent, perspective on the cost components of natural prices. In this representation natural prices comprise (i) the cost of means of production, $\mathbf{pA}$, (ii) the cost of maintaining the capitalist class at its conventional level of consumption, $\mathbf{pC}$, and (iii) the cost of labor, $\mathbf{l}w$.

Write equation (7) as an infinite series[4] to yield a 'complete reduction to dated quantities of labor',

$$\begin{aligned} \mathbf{p} &= \mathbf{l}w + \mathbf{l}(\mathbf{A} + \mathbf{C})w + \mathbf{l}(\mathbf{A} + \mathbf{C})^2 w + \cdots + \mathbf{l}(\mathbf{A} + \mathbf{C})^n w + \ldots \\ &= \sum_{n=0}^{\infty} \mathbf{l}(\mathbf{A} + \mathbf{C})^n w. \end{aligned}$$

[4]The infinite series converges on condition that matrix $\mathbf{A}+\mathbf{C}$ is productive, i.e. has a dominant eigenvalue less than one. If this condition does not hold then the level of capitalist consumption exceeds what is possible to technically reproduce.

In this series the profit-rate component of natural prices has been replaced by the labor cost of producing capitalist consumption goods. The wage rate is the only nominal variable that appears in the reduction. The reduction is therefore 'complete' or 'total' in the specific sense that it reduces *all* costs to labor costs.

The complete reduction reveals the additional labor supplied by workers to produce capitalist consumption goods at each 'successive stage' of the production of commodities. In comparison, Sraffa's reduction is incomplete and omits this labor because it leaves the profit-rate unreduced. Sraffa's reduction and the complete reduction are merely different representations of the same natural prices. One representation hides some labor performed, while the other reveals it.

We can directly observe capitalist consumption, $\mathbf{C}$, and measure it without knowledge of prices or the distribution of nominal income.

**Definition 1.** *'Nonstandard' labor-values (Wright, 2009, 2011a) are given by*

$$\tilde{\mathbf{v}} = \tilde{\mathbf{v}}\tilde{\mathbf{A}} + \mathbf{l}, \tag{8}$$

*where* $\tilde{\mathbf{A}} = \mathbf{A} + \mathbf{C}$ *is the technique augmented by capitalist consumption.*

Nonstandard labor-values measure total labor costs, i.e. the direct ($\mathbf{l}$), indirect ($\tilde{\mathbf{v}}\mathbf{A}$) *and* 'super-indirect' ($\tilde{\mathbf{v}}\mathbf{C}$) labor required to reproduce unit commodities, in circumstances of simple reproduction, where 'super-indirect' refers to the labor supplied to produce capitalist consumption.

Classical and nonstandard labor-values are answers to different questions. For example, Classical labor-values are 'technical' labor costs that allow productivity comparisons across time independent of the distribution of income (e.g., see especially (Flaschel, 2010, pt. 1)). Nonstandard labor-values, in contrast, are 'total' labor costs that include the 'tributary' or 'surplus' labor supplied to capitalists as a cost of production. Both kinds of measures are required for a labor theory of value.

For example, an immediate consequence of the complete reduction equation is that natural prices are proportional to nonstandard labor-values.

**Theorem 1.** *Given an economy with production prices,* $\mathbf{p} = \mathbf{p}\mathbf{A}(1 + r) + \mathbf{l}w$*, quantities,* $\mathbf{q} = \mathbf{q}\mathbf{A}^T + \mathbf{w} + \mathbf{c}$*, and zero saving (i.e. workers and capitalists spend what they earn), then*

$$\mathbf{p} = \tilde{\mathbf{v}}w,$$

*where* $\tilde{\mathbf{v}}$ *are nonstandard labor-values.*

*Proof.* From equation (7) $\mathbf{p} = \mathbf{p}\tilde{\mathbf{A}} + \mathbf{l}w = \mathbf{l}(\mathbf{I} - \tilde{\mathbf{A}})^{-1}w$. From the definition of nonstandard labor-values, $\tilde{\mathbf{v}} = \tilde{\mathbf{v}}\tilde{\mathbf{A}} + \mathbf{l} = \mathbf{l}(\mathbf{I} - \tilde{\mathbf{A}})^{-1}$. Hence $\mathbf{p} = \tilde{\mathbf{v}}w$. □

Sraffa's statement that prices and labor cost follow a 'simple rule' only in the special case of zero profit must therefore be qualified. The statement is correct for Classical labor-values, which measure technical costs of production, but false for a more general measure of labor cost that additionally includes the cost of reproducing the capitalist class. The period of 'waiting', which seems to exclude the possibility that labor costs can explain the structure of natural prices, is merely an artifact of an incomplete reduction. Natural prices, at all levels of the profit-rate, represent total labor costs, which in the context of simple reproduction are nonstandard labor-values. Sraffa's 'reduction to dated quantities of labor' fails to reveal this 'simple rule' because it is incomplete.

The Classical labor theory of value attempts to explain the structure of natural prices ('values') in terms of real costs of production, specifically labor costs. Sraffa, partly on the basis of his incomplete reduction, rejects this aspect of Classical theory. Nonetheless he attempts to circumvent some of the problems of the Classical labor theory in a remarkable but oblique manner.

## 4. The standard commodity

Consider situations A and B that share the same technology but differ in income distribution. Now, to consistently close the price system in both situations, we must specify an arbitrary *numéraire* equation, $\mathbf{p}\mathbf{d}^{\mathrm{T}} = 1$, where $\mathbf{d}$ is an arbitrarily chosen commodity bundle (this formulation includes the special case of setting one price to be unity, i.e. $p_i = 1$). Sraffa asks us to consider a measuring problem:

> The necessity of having to express the price of one commodity in terms of another which is arbitrarily chosen as standard [i.e., the *numéraire*], complicates the study of the price-movements which accompany a change in distribution. It is impossible to tell of any particular price-fluctuation whether it arises from the peculiarities of the commodity which is being measured or from those of the measuring standard (Sraffa, 1960, p. 18).

Since we define the price of the *numéraire* to be constant what can Sraffa mean by a price-fluctuation that 'arises from the peculiarities' of the *numéraire*? To answer this question

I borrow from formal analyses of Sraffa's standard commodity by Bellino (2004) and Baldone (2006).

Prices in Sraffa's equation (1) are a function of the wage and profit-rate. In the two situations, A and B, we have prices $\mathbf{p}_A = f(w_A, r_A)$ and $\mathbf{p}_B = f(w_A, r_B)$. The wage and profit-rate, prior to the choice of *numéraire*, are independent variables. Define $\mathbf{\Delta p} = \mathbf{p}_B - \mathbf{p}_A$, $\Delta r = r_B - r_A$ and $\Delta w = w_B - w_A$. The change in price of an arbitrary commodity bundle $\mathbf{d}$, from situation A to B, is then

$$\mathbf{\Delta p d}^{\mathrm{T}} = (1 + r_A + \Delta r)\mathbf{\Delta p A d}^{\mathrm{T}} + \Delta r \mathbf{p}_A \mathbf{A d}^{\mathrm{T}} + \Delta w \mathbf{l d}^{\mathrm{T}} \tag{9}$$

(following Baldone (2006); see Proposition 1 in the appendix).

Equation (9) is informative: the presence of the term $(1 + r_A + \Delta r)\mathbf{\Delta p A d}^{\mathrm{T}}$ tells us that, in general, the price of $\mathbf{d}$ changes partly due to changes in all other prices ($\mathbf{\Delta p}$) affecting the input cost of its means of production, i.e. $\mathbf{\Delta p A d}^{\mathrm{T}}$. In other words, the price of $\mathbf{d}$ fluctuates due to the transmission of relative price changes through its own 'peculiarities of production' or technical input requirements. The price of commodity bundle $\mathbf{d}$ is affected by, rather than isolated from, changes in the prices of all other commodities.

In consequence, if we happen to choose $\mathbf{d}$ as *numéraire*, i.e. $\mathbf{p d}^{\mathrm{T}} = 1$, which implies $\mathbf{\Delta p d}^{\mathrm{T}} = 0$, then the alteration in prices from A to B must satisfy the following constraint

$$0 = (1 + r_A + \Delta r)\mathbf{\Delta p A d}^{\mathrm{T}} + \Delta r \mathbf{p}_A \mathbf{A d}^{\mathrm{T}} + \Delta w \mathbf{l d}^{\mathrm{T}}.$$

Bellino (2004) calls this constraint the '*numéraire* effect' because the choice of *numéraire* affects how prices fluctuate, given the change in income distribution, $\Delta w$ and $\Delta r$. The *numéraire* itself imposes a constraint on $\mathbf{\Delta p}$ and therefore the standard in which prices are expressed affects the system it measures. Given an arbitrary *numéraire* $\mathbf{d}$ it's 'impossible to tell of any particular price fluctuation whether it arises from the peculiarities of the commodity which is being measured or from those of the measuring standard' (Sraffa, 1960, p. 18). This is Sraffa's measurement problem.

Sraffa therefore seeks a 'standard capable of isolating the price-movements [due to changes in the distribution of income] of any other product *so that they could be observed as in a vacuum*' (Sraffa, 1960, p. 18) (my emphasis). The 'vacuum' is an ideal situation that would remove the interfering effects of the *numéraire*'s own 'peculiarities of production'.

A 'measuring standard' independent of the price changes that occur between situation A and B would create such a 'vacuum'. Although such a standard 'would be no less susceptible than any other to rise or fall in price relative to other individual commodities; but we should know for certain that any such fluctuation would originate exclusively in the peculiarities of production of the commodity which was being compared with it, and not in its own' (Sraffa, 1960, p. 18).

The 'standard commodity' is Sraffa's answer to the measuring problem. The standard commodity[5] is the bundle of commodities **b** that satisfies

$$\lambda \mathbf{b} = \mathbf{b}\mathbf{A}^{\mathrm{T}} \tag{10}$$

where $\lambda$ is the dominant eigenvalue of technique $\mathbf{A}^{\mathrm{T}}$. The standard commodity **b** is therefore an eigenvector of $\mathbf{A}^{\mathrm{T}}$, and has the special property that, when multiplied by matrix $\mathbf{A}^{\mathrm{T}}$, it retains its proportions.

In economic terms, the production of 'the various commodities [that constitute bundle **b**] are produced in the same proportions as they enter the aggregate means of production [that is, $\mathbf{b}\mathbf{A}^{\mathrm{T}}$]', which 'implies that the rate by which the quantity produced exceeds the quantity used up in production is the same for each of them' (Sraffa, 1960, p. 20). Hence, if we consider the price of the standard commodity,

$$\begin{aligned} \lambda \mathbf{p}\mathbf{b}^{\mathrm{T}} &= \mathbf{p}\mathbf{A}\mathbf{b}^{\mathrm{T}} \\ \lambda &= \frac{\mathbf{p}\mathbf{A}\mathbf{b}^{\mathrm{T}}}{\mathbf{p}\mathbf{b}^{\mathrm{T}}}, \end{aligned} \tag{11}$$

then, regardless of prices **p**, the cost of production of the standard commodity, $\mathbf{p}\mathbf{A}\mathbf{b}^{\mathrm{T}}$, is always a constant fraction, $\lambda$, of its selling price. No matter how prices change this relationship always holds. In a sense, the 'peculiarities of production' of the standard commodity transmit cost price changes to the price of the output in an especially 'balanced' and invariant manner, a property explicitly inspired by Ricardo's notion of an 'average' commodity (Sraffa, 1960, p. 94).

But why does **b** constitute an invariable standard? Recall that Baldone's equation (9) describes the change in price of an arbitrary commodity bundle due to changes in

[5]For simplicity, and without loss of generality, I've ignored the normalization conditions that Sraffa imposes on his definition of the standard commodity.

income distribution. Let's now choose that arbitrary commodity to be Sraffa's standard commodity. Substitute (11) into (9):

$$\begin{aligned}\mathbf{\Delta p b}^{\mathrm{T}} &= (1 + r_A + \Delta r)\mathbf{\Delta p A b}^{\mathrm{T}} + \Delta r \mathbf{p}_A \mathbf{A b}^{\mathrm{T}} + \Delta w \mathbf{l b}^{\mathrm{T}} \\ &= (1 + r_A + \Delta r)\lambda \mathbf{\Delta p b}^{\mathrm{T}} + \Delta r \mathbf{p}_A \mathbf{A b}^{\mathrm{T}} + \Delta w \mathbf{l b}^{\mathrm{T}} \\ &= \frac{\Delta r \lambda \mathbf{p}_A \mathbf{b}^{\mathrm{T}} + \Delta w \mathbf{l b}^{\mathrm{T}}}{1 - \lambda(1 + r_A + \Delta r)}, \quad \lambda(1 + r_A + \Delta r) \neq 1, \end{aligned} \tag{12}$$

(the condition on the denominator is equivalent to the final profit-rate not reaching its theoretical maximum, i.e. $r_B \neq R$). The change in price of the standard commodity, in equation (12), is *independent of the change in prices*, $\mathbf{\Delta p}$, and only changes in virtue of the alteration in income distribution, $\Delta r$ and $\Delta w$. The variation of other prices does not affect the variation of the price of the standard commodity; the only relevant variable is the change in income distribution itself. Due to its special 'peculiarities of production' the standard commodity is isolated from the relative price changes that occur in the economy.

The standard commodity therefore meets Sraffa's requirement of invariance with respect to 'price-movements which accompany a change in distribution'. This is the fundamental meaning of the 'invariance' of the standard commodity: the '*numéraire* effect' is nullified and we have a measuring standard that does not affect the system it measures.

In general, the price of the standard commodity varies with income distribution.[6] The standard commodity's invariance is therefore completely different from the trivial 'invariance' of the *numéraire*, which, by construction, is constant (Bellino, 2004).

However, if we adopt the standard commodity as *numéraire* it confers a special property to the price system. Scale the standard commodity by a normalization factor, $\alpha\mathbf{b}$, where $\alpha = (1 - \lambda)/\mathbf{l b}^{\mathrm{T}}$ (in fact, Sraffa reserves the term 'standard commodity' for this normalized commodity bundle) and set the *numéraire* equation to $\alpha \mathbf{p b}^{\mathrm{T}} = 1$, then the maximum wage-rate is unity and

$$r = R(1 - w), \tag{13}$$

[6]Despite some claims in the literature (e.g., Baldone (2006) and also see Vienneau (2005) for a comprehensive review of claims regarding Sraffa's standard commodity) the price of the standard commodity is not invariant to changes in income distribution, except in special cases, such as an economy with gross output proportional to some scalar multiple of its standard commodity (i.e., an economy in Sraffa's 'standard proportions').

where $R$ is the maximum profit-rate (see Proposition 2 in the appendix). Equation (13) reveals a linear relationship between the profit and wage-rate: as $r$ increases from 0 to its maximum value $R$ then $w$ decreases from its maximum value 1 to 0. The standard commodity has 'render[ed] visible what was hidden' (Sraffa, 1960, p. 23), specifically the existence of a zero-sum distributional conflict between workers and capitalists that is logically independent of relative prices.[7]

## 5. The reduction to a 'variable quantity of labor'

Sraffa (1960, p. 31) considers the standard commodity 'a purely auxiliary construction' that can be 'displaced' by 'a more tangible measure for prices of commodities' which is 'the quantity of labor that can be purchased by the Standard net product' (Sraffa, 1960, p. 32), or, to use Smith's terminology, the 'labor commanded' (Smith, [1776] 1994) by the standard net product, i.e. its price divided by the wage rate. Denote this quantity of labor $\omega$; then, from equation (13),

$$\omega = \frac{\alpha \mathbf{p}\mathbf{b}^{\mathrm{T}}}{w} = \frac{R}{R-r}. \tag{14}$$

Sraffa (1960, p. 32) writes that 'all the properties of an "invariable standard of value" ... are found in a variable quantity of labor, which, however, *varies according to a simple rule which is independent of prices*: this unit of measurement increases in magnitude with the fall of the wage, that is to say with the rise of the rate of profits, so that, from being equal to the annual labor of the system when the rate of profits is zero, it increases without limit as the rate of profit approaches its maximum value at R' (my emphasis). (Sraffa normalizes the total labor of the system to unity; and hence $r = 0$ implies $\omega$ equals the 'annual labor'; but this normalization is not central to the construction.)

By adopting the standard commodity as *numéraire* 'in effect' (Sraffa, 1960, p. 32) we indirectly measure prices in terms of a variable quantity of labor, $\omega$, which is independent of the price changes that accompany a change in income distribution.

[7] Flaschel (2010, ch. 11) suggests we choose $\mathbf{p}\mathbf{n}^{\mathrm{T}} = 1$, where $\mathbf{n}$ is the net product, as the *numéraire* equation. We can then study conjectural variations in income distribution in the context of fixed income. Flaschel concludes, therefore, that Sraffa's standard commodity is superfluous. However, Flaschel's choice does not nullify the '*numéraire* effect' nor reveal the existence of a fixed *physical*, i.e. non-price, magnitude that breaks down into profit and wage income.

Why does Sraffa displace the standard commodity with $\omega$? Recall that, according to Sraffa's reduction equation, no 'simple rule' exists between natural prices and labor costs. In consequence, Classical labor-values cannot function as a price-independent, invariable standard of prices. However, Sraffa discovers, via the construction of the standard commodity, that in the specific case of changes in income distribution a (variable) quantity of labor is an invariable standard, and its variability follows a 'simple rule'.

Pasinetti (1977, p. 120) argues that the significance of the standard commodity is that we can 'treat the distribution of income independently of prices' and 'this possibility is not tied to the pure labor theory of value'. Equation (13) specifies how a given 'physical' quantity, $R$, determined by the objective conditions of production, breaks down into wage and profit income. Hence Sraffa's analysis preserves parts of the Classical 'surplus approach' to income distribution and separates it from the intractable contradictions of the labor theory. Sraffa's further step, of reducing the standard commodity to a quantity of labor, also reclaims, in attenuated form, aspects of the Classical theory of value, specifically the attempt to measure a given physical surplus *in terms of labor costs* and relate how that quantity of labor breaks down into wage and profit income.

## 6. The complete reduction to a 'variable quantity of labor'

Sraffa's route to a 'variable quantity of labor', $\omega$, requires we fix the profit-rate, $r$, which means that $\omega$ is irreducibly defined in terms of nominal, or monetary, phenomena. Sraffa's reduction of the standard commodity is incomplete in the specific sense that the 'variable quantity of labor' does not denote a real cost of production; it remains a 'labor commanded' measure of value. However, $\omega$ is completely reducible to a real cost:

**Theorem 2.** *Sraffa's 'variable quantity of labor', $\omega$, is the total labor cost of the standard commodity, $\alpha\mathbf{b}$; that is,*

$$\omega = \alpha\tilde{\mathbf{v}}\mathbf{b}^T. \tag{15}$$

*Proof.* Substitute $\mathbf{p} = \tilde{\mathbf{v}}w$ into equation (14) and the conclusion follows. □

Sraffa's 'variable quantity', therefore, is the direct, indirect and super-indirect labor required to reproduce the standard commodity given a technique, $\mathbf{A}$, and capitalist consumption, $\mathbf{c}$. The 'labor-commanded' measure is dual to a 'labor-embodied' measure.

Sraffa notes that his 'variable quantity of labor' varies from one to infinity as the profit-rate, $r$, varies from 0 to its maximum at $R$. We can examine this property from the point-of-view of conjectural variations in the distribution of real income, which are dual to the distribution of nominal income. Consider a given net product $\mathbf{n} = \mathbf{w}+\mathbf{c}$. Vary capitalist consumption between its minimum $\mathbf{c} = \mathbf{0}$ (such that $\mathbf{w} = \mathbf{n}$) and its maximum, $\mathbf{c} = \mathbf{n}$ (such that $\mathbf{w} = \mathbf{0}$).[8] At $\mathbf{c} = \mathbf{0}$ total labor costs collapse to Classical labor-values, $\tilde{\mathbf{v}} = \mathbf{v}$, and $\omega = 1$ (due to the choice of normalization). As $\mathbf{c}$ increases the capitalist class consumes a greater share of the net product, or surplus, and the labor-time required to produce their consumption increases. In consequence, the total labor cost of the standard commodity also increases. In the limit, capitalist consumption exhausts the whole surplus, leaving zero consumption for workers, at which point the economy cannot reproduce itself. Total labor costs, $\tilde{\mathbf{v}}$, approach infinity, indicating no quantity of labor is sufficient to reproduce unit commodities.

Sraffa embarks on a search for an invariable standard due to the 'necessity of having to express the price of one commodity in terms of another which is arbitrarily chosen as standard' (Sraffa, 1960, p. 18). The Classical labor theory of value proposed to 'express' prices in terms of an external standard but, as Sraffa's reduction equation demonstrates, Classical labor-values vary independently of prices and hence cannot be their measure. Prices, of 'necessity', must be measured in terms of other prices because an external standard does not exist. In consequence, we must address the problems of an internal standard or '*numéraire*'.

Sraffa defines a standard commodity, which has a price that functions as a fulcrum, and uses it to 'reach outside' the price system to the variable quantity of labor it commands in the market, which 'in effect' (Sraffa, 1960, p. 32) is an invariable standard. The complete reduction reveals that Sraffa's 'variable quantity' is, in fact, identical to the total labor cost of the standard commodity. Sraffa's 'variable quantity' is therefore an indirect or proxy reference to total labor costs, which is the external standard of prices missing from the Classical labor theory.

Sraffa (1960, p. 32) remarks, in the context of displacing the standard commodity, that 'it is curious that we should thus be enabled to use a standard without knowing what it consists of'. The standard commodity is a bridge from the premise that labor costs cannot

[8]More formally, we consider a monotonically increasing sequence $(\mathbf{c}_n)_{n=1}^{k}$ such that $\mathbf{c}_n \leq \mathbf{c}_{n+1}$, where $\mathbf{c}_n \in \{\mathbf{c} : \mathbf{c} \leq \mathbf{n}\}$ for all $n \in [1, k]$, $\mathbf{c}_0 = \mathbf{0}$, and $\mathbf{c}_k = \mathbf{n}$.

measure natural prices to the conclusion that a 'variable quantity of labor' is nonetheless an invariable standard. The bridge can be thrown away, and Sraffa's own analysis suggests it can, because the premise is mistaken.

Total labor costs, $\tilde{\mathbf{v}}$, immediately allow us to 'treat the distribution of income independently of prices' (Pasinetti, 1977, p. 120) because total labor costs are constituitively independent of prices and function as their measure. The problem of choosing an internal standard that nullifies the *numéraire* effect disappears as soon as we possess an external standard. The 'necessity' to express prices in terms of prices is not a necessity after all but the artifact of an incomplete reduction.

Total labor costs are entirely unaffected by 'price-movements which accompany a change in distribution'; in consequence, the standard commodity, and the labor it commands, can be displaced by total labor costs, which have 'all the properties of an "invariable standard of value"' as defined by Sraffa.

## 7. Sraffa and the Classical category-mistake

Classical natural prices count the money cost of reproducing the capitalist class as a component of the price of commodities, whereas Classical labor-values omit the labor cost of reproducing the capitalist class as a component of the labor-value of commodities. The dual systems of price and labor-value employ different cost accounting conventions. The price system measures total costs whereas the labor-value system measures partial costs.

A 'category-mistake' (Ryle, [1949] 1984) is the conceptual mistake of expecting some concept or thing to possess properties it cannot have. The Classical authors, such as Ricardo or Marx, commit a category-mistake when they expect their partial measure of labor cost, which is a function of the technical conditions of production, to be commensurate with a total measure of money costs, which is a function of technical conditions and the distribution of income. This expectation is necessarily confounded since only a total measure of labor cost can possess such a property. Hence Ricardo's problem of an invariable measure of value and Marx's transformation problem (Wright, 2011a).

Once we identify a category-mistake we can avoid it. For instance, the Classical antinomies dissolve when we adopt a more general definition of labor-value, which I have called 'nonstandard', which is the correct measure of total labor costs in circumstances of simple reproduction (see section 3 above and Wright (2009, 2011a, 2013)).

Sraffa's PCMC 'was explicitly designed to reconstruct the classical theory of value and distribution' (Kurz and Salvadori, 2000, p. 14) which, as Sraffa pointed out, had been 'submerged and forgotten since the advent of the "marginal" method' at the end of nineteenth century' (Sraffa, 1960, p. v). Sraffa demonstrates, via the remarkable construction of the standard commodity, that we can measure the physical surplus in terms of labor and relate that measure to actual money incomes. However, Sraffa's reconstruction does not identify or resolve the Classical category-mistake. In consequence, Sraffa's reductions to labor – the reduction of natural prices to 'dated quantities of labor' and the reduction of the standard commodity to a 'variable quantity of labor' – are incomplete. Sraffa's theory, like its Classical precursors, cannot sustain a concept of objective 'value' that reductively explains the structure of natural prices in terms of real costs of production. The post-Sraffian reconstruction of Classical economics therefore dispenses with an essential aim of a theory of economic value, which is to explain what the unit of account might measure or refer to.

In Sraffa's theory natural prices are reduced to an amalgam, the sum of quantities of labor and compound profits. The 'simple rule' that links total labor values, a physical real cost, to natural prices is absent. Sraffa's reconstruction of Classical economics is therefore incomplete.

## 8. Appendix

For clarity I include a complete numerical example of Theorems 1 and 2.

### *8.1. Numerical example of theorem 1*

We start with a given distribution of real income. The observed parameters are the technique $\mathbf{A} = \begin{bmatrix} 0.1 & 0.2 \\ 0.01 & 0.3 \end{bmatrix}$, direct labor coefficients $\mathbf{l} = \begin{bmatrix} 0.1 & 0.5 \end{bmatrix}$, real wage $\mathbf{w} = \begin{bmatrix} 0.5 & 0.2 \end{bmatrix}$, and capitalist consumption $\mathbf{c} = \begin{bmatrix} 0.05 & 0.01 \end{bmatrix}$.

Quantities, from equation (2), are $\mathbf{q} = \begin{bmatrix} 0.68 & 0.31 \end{bmatrix}$. The profit-rate consistent with this distribution of real income, from equation (5), is $r = 0.15$. Prices, from equation (1), are $\mathbf{p} = \begin{bmatrix} 0.12 & 0.81 \end{bmatrix} w$. The capitalist consumption matrix, from equation (6), is $\mathbf{C} =$

$\begin{bmatrix} 0.011 & 0.14 \\ 0.0021 & 0.028 \end{bmatrix}$. Nonstandard labor-values, from equation (8), are $\tilde{\mathbf{v}} = \begin{bmatrix} 0.12 & 0.81 \end{bmatrix}$. Hence $\mathbf{p} = \tilde{\mathbf{v}}w$, as per Theorem 1.

Alternatively, start with the observed technique, $\mathbf{A}$, direct labor coefficients, $\mathbf{l}$, and capitalist consumption matrix, $\mathbf{C}$. This is sufficient information to compute nonstandard labor-values, $\tilde{\mathbf{v}} = \begin{bmatrix} 0.12 & 0.81 \end{bmatrix}$, which then determine the structure of production-prices, $\mathbf{p}$.

### 8.2. *Numerical example of theorem 2*

Continuing our example: the 'standard commodity' for this economy, from eigenvector equation (10), is $\mathbf{b} = \begin{bmatrix} 1 & 1.048 \end{bmatrix}$ with dominant eigenvalue $\lambda = 0.31$. Sraffa reserves the term 'standard commodity' for the normalized bundle $\alpha\mathbf{b}$, where $\alpha = (1-\lambda)/\mathbf{l}\mathbf{b}^{\mathrm{T}} = 1.107$. Sraffa's 'variable quantity of labor', from equation (14), is then

$$\omega = \alpha\frac{\mathbf{p}\mathbf{b}^{\mathrm{T}}}{w} = 1.107 \times 0.968 = 1.07.$$

The nonstandard labor-value of the standard commodity, from equation (15), is $\tilde{\mathbf{v}}\alpha\mathbf{b}^{\mathrm{T}} = 1.07$, which equals Sraffa's 'variable quantity of labor', as per Theorem 2.

### 8.3. *Additional proofs*

**Proposition 1.** *Consider (i)* $\mathbf{p}_A\mathbf{d}^T = \mathbf{p}_A\mathbf{A}\mathbf{d}^T(1+r_A)+\mathbf{l}w_A$ *and (ii)* $\mathbf{p}_B\mathbf{d}^T = \mathbf{p}_B\mathbf{A}\mathbf{d}^T(1+r_B)+\mathbf{l}w_B$. *Define* $\Delta\mathbf{p} = \mathbf{p}_B - \mathbf{p}_A$, $\Delta w = w_B - w_A$ *and* $\Delta r = r_B - r_A$. *Then*

$$\Delta\mathbf{p}\mathbf{d}^T = (1+r_A+\Delta r)\Delta\mathbf{p}\mathbf{A}\mathbf{d}^T + \Delta r\mathbf{p}_A\mathbf{A}\mathbf{d}^T + \Delta w\mathbf{l}\mathbf{d}^T.$$

*Proof.* Subtract equation (i) from (ii):

$$\begin{aligned}
\Delta\mathbf{p}\mathbf{d}^{\mathrm{T}} &= \Delta\mathbf{p}\mathbf{A}\mathbf{d}^{\mathrm{T}} + \mathbf{p}_B\mathbf{A}\mathbf{d}^{\mathrm{T}}r_B - \mathbf{p}_A\mathbf{A}\mathbf{d}^{\mathrm{T}}r_A + \Delta w\mathbf{l}\mathbf{d}^{\mathrm{T}} \\
&= \Delta\mathbf{p}\mathbf{A}\mathbf{d}^{\mathrm{T}} + (\Delta\mathbf{p}+\mathbf{p}_A)\mathbf{A}\mathbf{d}^{\mathrm{T}}(\Delta r + r_A) - \mathbf{p}_A\mathbf{A}\mathbf{d}^{\mathrm{T}}r_A + \Delta w\mathbf{l}\mathbf{d}^{\mathrm{T}} \\
&= \Delta\mathbf{p}\mathbf{A}\mathbf{d}^{\mathrm{T}} + \Delta\mathbf{p}\mathbf{A}\mathbf{d}^{\mathrm{T}}\Delta r + \Delta\mathbf{p}\mathbf{A}\mathbf{d}^{\mathrm{T}}r_A + \mathbf{p}_A\mathbf{A}\mathbf{d}^{\mathrm{T}}\Delta r + \mathbf{p}_A\mathbf{A}\mathbf{d}^{\mathrm{T}}r_A - \mathbf{p}_A\mathbf{A}\mathbf{d}^{\mathrm{T}}r_A + \Delta w\mathbf{l}\mathbf{d}^{\mathrm{T}} \\
&= (1+r_A+\Delta r)\Delta\mathbf{p}\mathbf{A}\mathbf{d}^{\mathrm{T}} + \Delta r\mathbf{p}_A\mathbf{A}\mathbf{d}^{\mathrm{T}} + \Delta w\mathbf{l}\mathbf{d}^{\mathrm{T}}.
\end{aligned}$$

□

**Proposition 2.** *The* numéraire *equation* $\alpha\mathbf{p}\mathbf{b}^T = 1$, *where* $\alpha = (1-\lambda)/\mathbf{l}\mathbf{b}^T$, *implies*

$$r = R(1-w).$$

*Proof.* The *numéraire* equation $\alpha \mathbf{p}\mathbf{b}^{\mathrm{T}} = 1$ implies $\alpha \mathbf{p}\mathbf{A}\mathbf{b}^{\mathrm{T}} = \lambda$ by equation (11). Multiply Sraffa's price equation (1) by $\alpha \mathbf{p}\mathbf{b}^{\mathrm{T}}$:

$$
\begin{aligned}
\alpha \mathbf{p}\mathbf{A}\mathbf{b}^{\mathrm{T}}(1+r) + \alpha \mathbf{l}\mathbf{b}^{\mathrm{T}} w &= 1 \\
\lambda(1+r) + \alpha \mathbf{l}\mathbf{b}^{\mathrm{T}} w &= 1 \\
r &= \frac{1}{\lambda} - 1 - \frac{\alpha}{\lambda}\mathbf{l}\mathbf{b}^{\mathrm{T}} w \\
r &= R(1-w),
\end{aligned}
$$

by substituting for $\alpha$ and $R$ in the last step. □


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