ON NONSTANDARD LABOUR VALUES, MARX'S TRANSFORMATION PROBLEM AND RICARDO'S PROBLEM OF AN INVARIABLE MEASURE OF VALUE

1. **Introduction**

This essay is a critique of the classical labour theory of value when translated into the modern form of linear production theory. In many respects, the modern translation is an accurate and precise depiction of the deep conceptual structure of the classical theory; in some other respects it is not. Nonetheless the modern translation reproduces and clarifies two fundamental problems of the classical labour theory of value: Marx's 'transformation problem' (the contradiction between the law of value and uniform profits) and Ricardo's problem of an invariable measure of value (the lack of an objective 'measuring rod' to ground inter-temporal value comparisons).

The argument of this essay is that both problems derive from the same conceptual error, specifically the failure to properly specify replacement costs for a capitalist economy. The transformation problem and the problem of an invariable measure of value are both symptoms of this underlying labour-cost accounting error.

Once this error is corrected the classical problems are resolved. The theoretical result is the logical possibility of a 'nonstandard' labour theory of value.

2. **Self-replacing equilibrium**

The simplest case in which the classical problems manifest is a multi-sector equilibrium model of simple reproduction with uniform profits in all branches of production.

Consider an abstract capitalist economy in a self-replacing state. The technique is a non-negative $n \times n$ matrix of inter-sector coefficients, $\mathbf{A} = [a_{i,j}]$. Each $a_{i,j} \geq 0$ is the quantity of commodity i directly required to output 1 unit of commodity j. Assume there is a column vector $\mathbf{x} \in \mathrm{R}^n_+$ such that $\mathbf{x} > \mathbf{Ax}$; that is, the technique is productive. The direct labour coeficients are a $1 \times n$ row vector, $\mathbf{l} = [l_i]$. Each $li > 0$ is the quantity of labour directly required to output 1 unit of commodity i.

As the technique is productive the economy produces a net product, a $1 \times n$ row vector, $\mathbf{n} = [n_i]$. Each $n_i \geq 0$ is the quantity of commodity i available for consumption after capital stocks are replaced. The real wage is a $1 \times n$ row vector, $\mathbf{w} = [w_i]$. Each $w_i \geq 0$ is the quantity of commodity i consumed by workers. Capitalist consumption is a $1 \times n$ row vector, $\mathbf{c} = [c_i]$. Each $c_i \geq 0$ is the quantity of commodity i consumed by capitalists. No capital accumulation takes place. Hence the whole net product is distributed to workers and capitalists for consumption; that is $\mathbf{n} = \mathbf{w} + \mathbf{c}$.

The variables $\mathbf{A}$, $\mathbf{l}$, $\mathbf{w}$ and $\mathbf{c}$ are given data. The data satisfy a physical quantities equation

(1) $$\mathbf{q} = \mathbf{qA}^{\mathrm{T}} + \mathbf{n},$$

where $\mathbf{q} = [q_i]$ is a $1 \times n$ row vector of gross output. Each $q_i > 0$ is measured in units of commodity i.

The data also satisfy a price equation

$$\mathbf{p} = (\mathbf{pA} + \mathbf{l}w)(1 + r), \quad (2)$$

where $\mathbf{p} = [p_i]$ is a $1 \times n$ row vector of prices. Each p_i is measured in money units per unit of commodity-type i. $w \geq 0$ is the money wage rate, measured in money units per unit of labour-time. The 'rate of profit', $r \geq 0$, is a ratio of money amounts that scales input costs. Equation(2) defines profit-equalising prices of production ('competitive prices'). Prices equal input costs plus profit.

Define $t = 0$ as the 'date' of the current period and $t - 1$ as the date of the previous period. Consider production at $t = 0$. A stock of capital commodities, $\mathbf{qA}^{\mathrm{T}}$, produced in period $t - 1$, is used-up. A stock of consumption goods, $\mathbf{n}$, produced in period $t - 1$, is consumed. Gross output, $\mathbf{q}$, is produced. A part of the gross output, $\mathbf{qA}^{\mathrm{T}}$, replaces the used-up capital and is input to period $t + 1$. A part, $\mathbf{n}$, replaces the used-up consumption goods, and is input to period $t + 1$. Let $\mathbf{q}(t)$ denote the gross output of the economy at t. Then $\mathbf{q}(t) = \mathbf{q}(t + 1)$ for all t.

In this hypothetical state of self-replacing equilibrium the economy continually reproduces its own material conditions of production.[1]

[1] The reader interested in formal mathematical proofs of the propositions in this paper may consult reference (WRIGHT 2007).

3. Standard labour values

DMITRIEV (1868-1913) was the? first economist to translate the classical concept of 'labour embodied' into a mathematical formula for the actual calculation of the labour-value of commodities (NUTI 1974, DMITRIEV 1974). Dmitriev's formula is now standard (e.g., SRAFFA (1960), SAMUELSON (1971), PASINETTI (1977), STEEDMAN (1981)).

The $1 \times n$ vector $\mathbf{v}$ of *standard labour-values* is defined by the equation

$$\mathbf{v} = \mathbf{vA} + \mathbf{l}. \quad (3)$$

Labour-values are the sum of 'dead' or indirect labour 'embodied' in means of production ($\mathbf{vA}$) plus an addition of 'living' ordirect labour ($\mathbf{l}$). Since labour-values are a function of the current technique $\mathbf{A}$ and direct labour costs $\mathbf{l}$ they measure the 'total sum of the labour directly and indirectly expended on the production of any product *under present-day production conditions*' independent of any 'historical digressions' regarding the past state of the economy (DMITRIEV (1974), pp. 43-44). For example, MARX ([1887] 1954) writes, 'the value of a commodity is determined not by the quantity of labour actually realized in it, but by the quantity of living labour necessary for its production. A commodity represents, say 6 working hours. If an invention is made by which it can be produced in 3 hours, the value, even of the commodity already produced, falls by half. It represents now 3 hours of social labour instead of the 6 formerly necessary. It is the quantity of labour required for its production, not the realized form of that labour, by which the amount of the value of a commodity is determined.'

We can interpret the meaning of equation (3) in a number of different ways. For ease of exposition I will present the core argument of this essay in terms of a conventional 'dated' interpretation; the conclusions, however, are independent of this choice. Rearrange equation (3) to get

$$\mathbf{v} = \mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}, \qquad (4)$$

where $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1} = [\alpha_{i,j}]$ is the Leontief inverse. Replace the Leontief inverse $\mathbf{L}$ by its power-series representation to get

$$\begin{aligned} \mathbf{v} &= \mathbf{l}\sum_{n=0}^{\infty} \mathbf{A}^n \\ &= \mathbf{l}(\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \cdots + \mathbf{A}^n + \ldots). \end{aligned} \qquad (5)$$

Interpret each term in series (5) as representing production that occurred at a particular 'date'. The infinite series then represents a 'process' that occurs in logical time. The first term, $\mathbf{I}$, represents the final output of unit commodities at $t = 0$; the second term, $\mathbf{A}$, represents the heterogeneous inputs used-up by each sector at $t = -1$ in order to produce unit commodities as output at t = 0; and the third term, $\mathbf{A}^2$, represents the inputs used-up at $t = -2$ to output the stock of commodities used at $t = -1$; and so forth, back in 'time'. The ith column of the matrix $\mathbf{A}^n$ represents the stocks of commodities productively consumed by each sector at time $t = -n$. Since the Leontief inverse is the sum of every term in the series each $\alpha_{i,j}$ representsthetotalphysicalquantity of the ith commodity used-up directly and indirectly in order to obtain the availability of 1 physical unit of the jth commodity as a component of net output (PASINETTI (1977), p. 64).

Labour-values are formed by multiplying each term of the series by the direct labour coeficients; that is,

$$\mathbf{v} = \mathbf{l} + \mathbf{lA} + \mathbf{lA}^2 + \cdots + \mathbf{lA}^n + \ldots .$$

Each $\mathbf{lA}^n$ term is a vector that represents the living labour used-up in each sector at time $t = -n$. So the dated interpretation describes aprocess that extends back in time, until, in the limit all commodity stocks are ultimately reduced to labour alone. Each labour-value v_i is there fore the 'vertical integration' (PASINETTI 1977) of the labour used-up at successive stages in order to output 1 unit of commodity i.

For example, SRAFFA writes that the labour embodied in a commodity is 'the sum of a series of terms when we trace back the successive stages of the production of the commodity' (SRAFFA 1960), p. 89); and SAMUELSON writes that 'the accuracy of this result can be verified by going back in time to add up the dead labour needed at *all* the previous stages' (SAMUELSON 1971) (emphasis in original).

An important property of the dated interpretation of labour-values has not been suffciently examined: at every successive stage an output is produced but it is productively invested rather than consumed by workers and capitalists.

To see this multiply series (5) by agross product $\mathbf{q}$ to derive its total labour-value,

$$\mathbf{vq}^{\mathrm{T}} = \mathbf{lq}^{\mathrm{T}} + \mathbf{lAq}^{\mathrm{T}} + \mathbf{lA}^2\mathbf{q}^{\mathrm{T}} + \cdots + \mathbf{lA}^n\mathbf{q}^{\mathrm{T}} + \ldots . \qquad (6)$$

$\mathbf{q}^{\mathrm{T}}$ is produced at $t = 0$ with total direct labour $\mathbf{lq}^{\mathrm{T}}$ and input commodities $\mathbf{Aq}^{\mathrm{T}}$. The input commodities $\mathbf{Aq}^{\mathrm{T}}$ are produced at $t = -1$ with total direct labour $\mathbf{lAq}^{\mathrm{T}}$ and input commodities, $\mathbf{A}^2\mathbf{q}^{\mathrm{T}}$; and so forth, back in time. Hence the gross output at date $t \in [0, -\infty)$is

$$\mathbf{q}(t) = \mathbf{q}(\mathbf{A}^{\mathrm{T}})^{|t|}.$$

Clearly, $\mathbf{q}(t) \neq \mathbf{q}(t-1)$ for all t. Hence, during the 'successive stages of the production' of $\mathbf{q}$ the economy is *not* in a state of self-replacing equilibrium. In fact, since $\mathbf{A}$ is a productive matrix with a dominant eigenvalue positive but less than 1, $\mathbf{q}(t) > \mathbf{q}(t-1)$ and the economy is *growing*.

The reason for this difference is simple. The dated interpretation of labour-value posits a *hypothetical process* occurring in logical time that terminates in production at scale $\mathbf{q}$, whereas the concept of self-replacing equilibrium describes an *actual process* in which the economy is in a *steady state* of continual reproduction at scale $\mathbf{q}$. Call the hypothetical process the 'process of replacement', or simply 'replacement'. Labour-values are therefore the *replacement costs* of unit commodities measured in units of labour-time: they represent how much labour is used-up to produce commodities 'from scratch'. But the process of replacement does not in fact occur. It is a counter-factual interpretation of a property of the *current state*, or 'present-day production conditions', of the economy.

We can gain adeeper understanding of the meaning of labour-values by examining the process of replacement more closely. During replacement of commodities $\mathbf{q}$ production at t uses-up and replaces the inputs, $\mathbf{q}(t-1)$, and generates an additional netproduct or surplus $\mathbf{n}(t)$; that is we can write the relationship between successive stages as

$$\mathbf{q}(t) = \mathbf{q}(t-1) + \mathbf{n}(t). \tag{7}$$

(At $t = 0$ equation (7) reduces to $\mathbf{q}(0) = \mathbf{q}(0)\mathbf{A}^T + \mathbf{n}(0)$, which is equilibrium quantity equation (1).)

In self-replacing equilibrium the net product $\mathbf{n}$ is consumed by workers and capitalists. During the process of replacement, however, the sequence of net products, $(\mathbf{n}(t))$, is *not* consumed by workers and capitalists; rather, the net products are reinvested and function as means of production for the next round of production. The economy grows during the hypothetical process of replacement because both

workers and capitalists abstain from consumption.[2] (Consider if households did not abstain from the consumption of the net product $\mathbf{n}(t)$ at $t < 0$. Then $\mathbf{q}(t) = \mathbf{q}(t-1)$, the economy does not grow, and replacement terminates at scale $\mathbf{q}(t) < \mathbf{q}(0) = \mathbf{q}$.)

To produce $\mathbf{q}$ requires a total quantity of direct labour $L = \mathbf{q}\mathbf{l}^T$. From(1), $\mathbf{n}^T = (\mathbf{I} - \mathbf{A})\mathbf{q}^T$ and from(4), $\mathbf{v} = \mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}$. Hence $\mathbf{v}\mathbf{n}^T = \mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}(\mathbf{I} - \mathbf{A})\mathbf{q}^T = \mathbf{l}\mathbf{q}^T$. So the labour-value of the net product equals the total direct labour (or length of the working day):

$$\mathbf{v}\mathbf{n}^T = \mathbf{l}\mathbf{q}^T = L. \tag{8}$$

Equation (8) is a tautology satisfied by standard labour--values; call it the 'net value equality'.

Pasinetti (1980) interprets the net value equality as expressing two different ways of classifying, or disaggregating, the total labour L. The expression $L = \mathbf{l}\mathbf{q}^T$ classifies the total labour 'according to the criterion of the industry in which [it is] required'. The expression $L = \mathbf{v}\mathbf{n}^T$ classifies the total labour 'according to the criterion of the vertically integrated sector for which [it is] directly and indirectly required'.

The net value equality can also be interpreted in terms of the dated interpretation. During replacement of $\mathbf{q}$ the net product $\mathbf{n}(t)$ is *not* consumed by households at *every* stage. An amount of direct labour $\mathbf{l}\mathbf{n}^T(t)$ is therefore *not performed* due to abstinence. The total labour notperformed during replacement is $\mathbf{l}\sum_{t=0}^{-\infty}\mathbf{n}^T(t) = \mathbf{l}\sum_{t=0}^{-\infty}\mathbf{q}^T(t) - \mathbf{q}^T(t-1) = \mathbf{l}(\mathbf{I} - \mathbf{A})(\sum_{n=0}^{\infty}\mathbf{A}^n)\mathbf{q} = \mathbf{l}(\mathbf{I} - \mathbf{A})(\mathbf{I} - \mathbf{A})^{-1}\mathbf{q} = \mathbf{l}\mathbf{q}^T$.

Hence $\mathbf{l}\mathbf{q}^T$ is the total labour not performed when $\mathbf{n}$ is not replaced. On the other hand, $\mathbf{v}\mathbf{n}^T$ is the total labour

[2] Of course workers and capitalists are in fact consuming in the steady state.

performed if n is replaced. The net value equality simply states that the labour used-up if households consume is equal to the labour saved if households abstain.[3]

The claim that **v** counts *all* the labour of previous stages must therefore be qualified. The standard definition adds up all the labour of previous stages on the assumption that workers and capitalists abstain during replacement. But if this assumption is relaxed then v will not count all the labour of previous stages.

On what grounds is this assumption essential to the calculation of labour-values? We will show that it is not essential. But more importantly we will show that the assumption is *incorrect* given the theoretical intent of measuring replacement costs in terms of labour time.

4. **The circular flow**

Consider the consequences of assuming that capitalist households consume, rather than abstain, during replacement. How is capitalist consumption synchronised with production?

The given data together with the quantity and price equations necessarily determine a circular flow representation of the economy that specifies the input-output relations between households and production. Quantity equation (1)

[3] Alternatively, assume labour-values **v** and the total labour force L is fixed but the net product **n** is a free variable. Then the net value equality, $\mathbf{vn}^{\mathrm{T}} = L$, is a hyper-plane equation that represents the net product possibility frontier. Each point on the surface of the hyper-plane is a possible composition of the net product n that may be produced given current technology and labour resources. Ratios of labour-values, $w_{i,j} = v_i/v_j$, represent marginal rates of transformation ('trade-off possibilities') in the net product between commodities i and j.

determines the gross product **q**. The real wage rate is $\bar{\mathbf{w}} = \mathbf{w}/L$. In self-replacing equilibrium the money wage rate exactly covers the cost of the real wage, hence

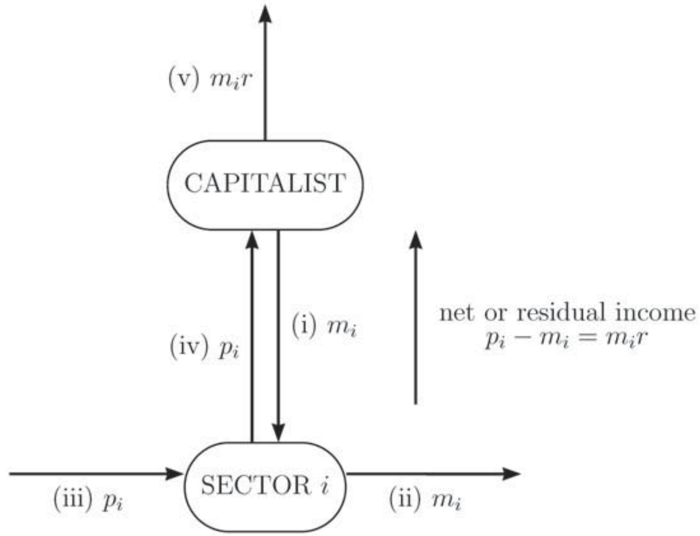

FIGURE 1. **Monetary exchange between capitalist households and production activities**. (i) Capitalist households supply m_i money-capital to the sector (or firm); (ii) the m_i is spent on input commodities and labour; (iii) production occurs and unit output is sold for p_i revenue; (iv) the revenue p_i is transferred to the capitalist owner(s). A part m_i funds the next period of production. The residual $m_i r$, where r isthe rate of profit, is profit income. (v) The profit income $m_i r$ is spent in consumption good markets.

$\omega = \mathbf{p}\bar{\mathbf{w}}^{\mathrm{T}}$. Substituting for ω inprice equation (2) gives

$$\begin{aligned} \mathbf{p} &= (\mathbf{pA} + \mathbf{p}\bar{\mathbf{w}}^{\mathrm{T}}\mathbf{l})(1+r) \\ &= \mathbf{pA}^{+}(1+r), \end{aligned} \quad (9)$$

where $\mathbf{A}^{+} = \mathbf{A} + \bar{\mathbf{w}}^{\mathrm{T}}\mathbf{l}$ is the technique augmented by workers consumption. Let $\lambda_* = 1/(1+r)$ and rearrange to get

$$\mathbf{pA}^{+} = \lambda_* \mathbf{p}. \quad (10)$$

The maximum eigenvalue solution of (10) yields λ_* and hence the profit rate $r = (1/\lambda_*) - 1$. $\mathbf{p}$ is the left-hand eigenvector of $\mathbf{A}^+$ associated with λ_* and is determined up to the choice of *numéraire*.

Assume non-commodity money, such as paper currency. Money, in the hands of capitalists, functions as capital since its advance to production commands a return. In contrast, money, in the hands of consumers, either workers or capital ists spending for personal consumption, does not command a return and merely functions as means of exchange. We shall use the term 'money-capital' to denote

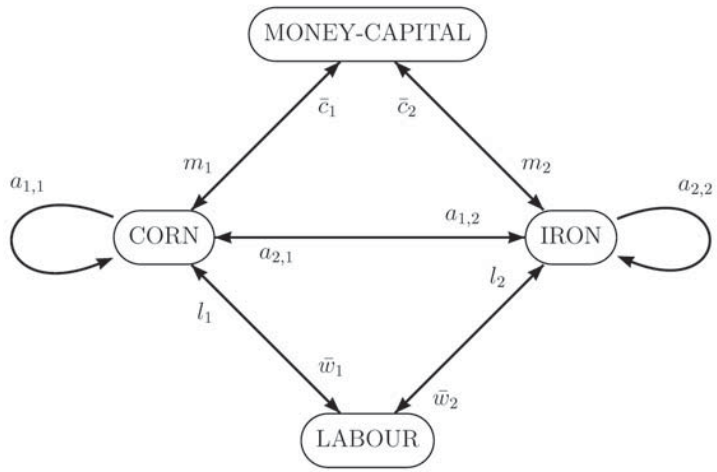

FIGURE 2. **The synchronisation of capitalist consumption with production in a 2-commodity circular flow.**

money when it functions as capital.[4] Capitalists supply money--capital to cover the input costs of the period of production. Figure 1 describes the monetary transfers between capitalist households and production activities.

[4] Money can function both as money-capital and means of exchange during its circuit.

Prices equal unit input costs $\mathbf{m}$ plus the profit mark-up, $\mathbf{p} = \mathbf{m} + \mathbf{m}r = \mathbf{m}(1 + r)$; hence $\mathbf{m} = 1/(1 + r)\mathbf{p}$. The total quantity of money-capital advanced in the production period is $M = \mathbf{q}\mathbf{m}^{\mathrm{T}}$.[5] The capitalist consumption rate, that is the quantities of commodities consumed per unit of money--capital advanced, is therefore $\bar{\mathbf{c}} = \mathbf{c}/M$.

Money-capitalis money that returns to the capitalist with a profit increment. 1 unit of money-capital advanced during the production period generates a profit income of $1r$ units of money, where r is equivalently the rate of profit or the price of money-capital. r is measured in money units per unit of money-capital (a pure number for a given time period). In self-replacing equilibrium profit income is spent on consumption goods $\mathbf{c}$. Hence the price of money-capital equals the rate of capitalist expenditure on consumption goods,

(11) $$\mathbf{r} = \mathbf{p}\bar{\mathbf{c}}^{\mathrm{T}}.$$

(Equivalence (11) is proved in the appendix). The equality of the price of money-capital and the cost of capitalist consumption, $r = \mathbf{p}\bar{\mathbf{c}}^{\mathrm{T}}$, is the counterpart of the equality of the price of labour and the cost of the real wage, $w = \mathbf{p}\bar{\mathbf{w}}^{\mathrm{T}}$.

For example, consider a 2-commodity economy that produces corn and iron where $\mathbf{A} = \begin{bmatrix} 0.5 & 0.2 \\ 0.6 & 0.4 \end{bmatrix}$, $\mathbf{l} = [\ 10\ 1\ ]$, $\mathbf{w} = [\ 2\ \ 0\ ]$, $\mathbf{c} = [\ 1\ \ 0\ ]$. From (1) the gross product is $\mathbf{q} = [\ 10\ 10\ ]$, $L = 110$ and the real wage rate is $\bar{\mathbf{w}} = [\ 0.018\ \ 0\ ]$.

[5] Note that this statement is independent of the total stock of paper currency required to circulate commodities.

$$\mathbf{A}^{+} = \mathbf{A} + \bar{\mathbf{w}}\mathbf{l}^{\mathrm{T}} = \mathbf{A} + \begin{bmatrix} 0.182 & 0.018 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0.682 & 0.218 \\ 0.6 & 0.4 \end{bmatrix}.$$

The eigenvalue equation $\mathbf{pA}^{+} = \lambda\mathbf{p}$ yields the characteristic equation $\lambda^2 - 1.082\lambda + 0.142 = 0$. The dominant root is $\lambda_* = 0.929$; hence the rate of profit $r = 0.076$ or 7.6%. Solving the eigenvector equation $\mathbf{p}(\mathbf{A}^{+} - \lambda_*\mathbf{I}) = \mathbf{0}$ yields $\mathbf{p} = p_1[\,1 \quad 0.412\,]$ where p_1 is the *numéraire*. Thus $\mathbf{m} = 1/(1 + r)\mathbf{p} = p_1[\,0.929 \quad 0.383\,]$, $M = \mathbf{qm}^{\mathrm{T}} = 13.12p_1$ and the capitalist consumption rateis $\bar{\mathbf{c}} = (1/p_1)[\,0.076 \;\, 0\,]$. The price of money-capital equals the price of capitalist consumption, $r = \mathbf{p}\bar{\mathbf{c}}^{\mathrm{T}} = 0.076$ money units per unit of money-capital. Figure 2 graphs the circular flow for this 2-commodity example.

Cost prices, **m**, capitalist consumption per unit of money--capital advanced, $\bar{\mathbf{c}}$, and the real wage rate, $\bar{\mathbf{w}}$, are all dependent variables of the definition of self-replacing equilibrium.

5. Nonstandard labour values

Consider the matrix of capitalist consumption coefficients, $\mathbf{B} = \bar{\mathbf{c}}^{\mathrm{T}}\,\mathbf{m} = [b_{i,j}]$, where each $b_{i,j}$ is the quantity of commodity i capitalists consume per unit output of commodity j. Define the technique augmented by capitalist consumption as $\tilde{\mathbf{A}} = \mathbf{A} + \bar{\mathbf{c}}^{\mathrm{T}}\mathbf{m} = [\tilde{a}_{i,j}]$.[6] $\tilde{a}_{i,j} > 0$ is the quantity of commodity i, including capitalist consumption, directly used-up per unit output of j.

In the case of simple reproduction with a uniform rate of profit the $1 \times n$ vector $\tilde{\mathbf{v}}$ of *nonstandard labour values* is defined by the equation[7]

$$\tilde{\mathbf{v}} = \tilde{\mathbf{v}}\tilde{\mathbf{A}} + \mathbf{l}. \tag{12}$$

Nonstandard labour-values are the sum of dead labour 'embodied' in means of production and capitalist consumption goods ($\tilde{\mathbf{v}}\tilde{\mathbf{A}}$) plus an addition of living labour (**l**). Rearrange equation (12) to get

$$\tilde{\mathbf{v}} = \mathbf{l}(\mathbf{I} - \tilde{\mathbf{A}})^{-1}, \tag{13}$$

where $\tilde{\mathbf{L}} = (\mathbf{I} - \tilde{\mathbf{A}})^{-1} = [\tilde{\alpha}_{i,j}]$. Each $\tilde{\alpha}_{i,j}$ represents the total physical quantity of the ith commodity used-up directly and indirectly in order to obtain the availability of 1 physical unit of the jth commodity as a component of the real wage. Each nonstandard labour-value $\tilde{v}_i$ is therefore the 'vertical integration' of all the labour used-up at successive stages, including that required to produce capitalist consumption goods, in order to output 1 unit of commodity i.

Equation (13) can also be given a 'dated' interpretation. Replace $\tilde{\mathbf{L}}$ by its power-series representation to get

$$\tilde{\mathbf{v}} = \mathbf{l} + \mathbf{l}(\mathbf{A} + \bar{\mathbf{c}}^{\mathrm{T}}\mathbf{m}) + \mathbf{l}(\mathbf{A} + \bar{\mathbf{c}}^{\mathrm{T}}\mathbf{m})^2 + \cdots + \mathbf{l}(\mathbf{A} + \bar{\mathbf{c}}^{\mathrm{T}}\mathbf{m})^t + \ldots \,. \tag{14}$$

[6] The technique augmented by capitalist consumption is a convenient representation of how commodities, including capitalist consumption goods, are used-up during the period of production. But it does not imply that capitalist consumption goods are direct inputs to firm production (c.f. Figure 2). The need to 'collapse' capitalist consumption into the technique is merely an artefact of the 'open' linear production model adopted by LEONTIEF and SRAFFA in which final demand is not fully specified. For example, the technique augmented by capitalist consumption is equivalent to a 'closed' representation of the economy in which worker and capitalist households are distinct sectors of a higher dimensional input-output matrix (WRIGHT 2007).

[7] See the appendix for an analysis of nonstandard labour-values in the case of proportionate growth. In general nonstandard labour-values are a property of the social accounting matrix whereas standard labour-values are a property of the technique alone.

The first term, $\mathbf{l}$, is the direct labour applied at $t = 0$ to produce unit commodities; the second term, $\mathbf{l}\tilde{\mathbf{A}}$, is the direct labour applied at $t = -1$ to produce the stock of commodities, including capitalist consumption goods, used-up at $t = 0$; and so forth, back in time. Hence the direct labour used-up at successive stages to maintain the capitalist class is counted as a real cost of production during the nonstandard process of replacement.

At every stage a net product is produced but, in contrast to standard labour values, only the part *net* of capitalist consumption is productively invested.

To see this multiply series (14) by agross product $\mathbf{q}$ to derive its total labour-value,

(15) $$\tilde{\mathbf{v}}\mathbf{q}^{\mathrm{T}} = \mathbf{l}\mathbf{q}^{\mathrm{T}} + \mathbf{1}\tilde{\mathbf{A}}\mathbf{q}^{\mathrm{T}} + \mathbf{l}\tilde{\mathbf{A}}^{2}\mathbf{q}^{\mathrm{T}} + \cdots + \mathbf{l}\tilde{\mathbf{A}}^{n}\mathbf{q}^{\mathrm{T}} + \ldots .$$

Gross output at date $t \in [0, -\infty)$ is

$$\tilde{\mathbf{q}}(t) = \mathbf{q}(\tilde{\mathbf{A}}^{\mathrm{T}})^{|t|}.$$

Clearly, $\tilde{\mathbf{q}}(t) \neq \tilde{\mathbf{q}}(t-1)$ for all t. So, like the dated interpretation of standard labour-values, the economyisgrowing. Butcapitalists consumeduringtheprocess of replacement. Hence $\tilde{\mathbf{q}}(t) > \mathbf{q}(t)$ because the nonstandard dated gross output additionally replaces used-up capitalist consumption goods.

During replacement the sequence of net products is $(\mathbf{w}(t))$ not $(\mathbf{n}(t))$ because only workers abstain from consumption. Production at t replaces the means of production and capitalist consumption goods, $\tilde{\mathbf{q}}(t-1)$, and produces a net product or surplus $\mathbf{w}(t)$; that is

(16) $$\tilde{\mathbf{q}}(t) = \tilde{\mathbf{q}}\ (\mathrm{t}-1) + \mathbf{w}(t).$$

(At $t = 0$ equation (16) reduces to $\tilde{\mathbf{q}}(0) = \tilde{\mathbf{q}}(0)\tilde{\mathbf{A}}^{\mathrm{T}} + \mathbf{w}(0) = = \tilde{\mathbf{q}}(0)\mathbf{A}^{\mathrm{T}} + \mathbf{c}(0) + \mathbf{w}(0)$, which is equilibrium quantity equation (1).)

Non standard labour-values do not satisfy net value equality (8). From (1),

$$\begin{aligned}\mathbf{w}^{\mathrm{T}} &= (\mathbf{I} - \mathbf{A})\mathbf{q}^{\mathrm{T}} - \mathbf{c}^{\mathrm{T}}\\ &= (\mathbf{I} - \mathbf{A})\mathbf{q}^{\mathrm{T}} - M\bar{\mathbf{c}}^{\mathrm{T}}\\ &= (\mathbf{I} - \mathbf{A})\mathbf{q}^{\mathrm{T}} - \bar{\mathbf{c}}^{\mathrm{T}}\mathbf{m}\mathbf{q}^{\mathrm{T}}\\ &= (\mathbf{I} - (\mathbf{A} + \bar{\mathbf{c}}^{\mathrm{T}}\mathbf{m}))\mathbf{q}^{\mathrm{T}}\\ &= (\mathbf{I} - \tilde{\mathbf{A}})\mathbf{q}^{\mathrm{T}}.\end{aligned}$$

And from (13), $\tilde{\mathbf{v}} = \mathbf{l}(\mathbf{I} - \tilde{\mathbf{A}})^{-1}$ Hence $\tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}} = \mathbf{l}(\mathbf{I} - \tilde{\mathbf{A}})^{-1}(\mathbf{I} - \tilde{\mathbf{A}})\mathbf{q}^{\mathrm{T}} = \mathbf{l}\mathbf{q}^{\mathrm{T}}$. So the nonstandard labour-value of the real wage equals the total direct labour (or length of the working day):

(17 $$\tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}} = \mathbf{l}\mathbf{q}^{\mathrm{T}} = L.$$

Equation (17) is a tautology satisfed by nonstandard labour--values; calli t the 'wage value equality'.

The wage value equality can alsobe understood in terms of the dated interpretation. During nonstandard replacement of $\mathbf{q}$ a net product $\mathbf{w}(t)$ is *not* consumed by workers at *every* stage. Hence, an amount of direct labour $\mathbf{l}\mathbf{w}^{\mathrm{T}}(t)$ is *not performed* due to the non-replacement of $\mathbf{w}(t)$. The total labour not performed during nonstandard replacement is $\mathbf{l}\sum_{t=0}^{-\infty}\mathbf{w}^{\mathrm{T}}(t) = \mathbf{l}\sum_{t=0}^{-\infty}\tilde{\mathbf{q}}^{\mathrm{T}}(t) - \tilde{\mathbf{q}}^{\mathrm{T}}(t-1) = \mathbf{l}(\mathbf{I} - \tilde{\mathbf{A}})(\sum_{n=0}^{-\infty}\tilde{\mathbf{A}}^{n})\mathbf{q}^{\mathrm{T}} = = \mathbf{l}(\mathbf{I} - \tilde{\mathbf{A}})(\mathbf{I} - \tilde{\mathbf{A}})^{-1}\mathbf{q}^{\mathrm{T}} = \mathbf{l}\mathbf{q}^{\mathrm{T}}$. Hence $\mathbf{l}\mathbf{q}^{\mathrm{T}}$ is the total labour not performed when $\mathbf{w}$ is not replaced. On the other hand, $\tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}}$ is the total labour performed if $\mathbf{w}$ is replaced. The wage value equality simply states that the labour used-up if workers consume is equal to the labour saved if workers abstain.[8]

Definition (12) can be written $\tilde{\mathbf{v}} = \tilde{\mathbf{v}}\mathbf{A} + (\tilde{\mathbf{v}}\bar{\mathbf{c}}^{\mathrm{T}})\mathbf{m} + \mathbf{l}$, where the scalar $\tilde{\mathbf{v}}\bar{\mathbf{c}}^{\mathrm{T}}$ is the labour-value of money-capital, which measures how much direct and indirect labour is used-up per unit of money-capital advanced. Hence nonstandard labour-values are the sum of dead labour 'embodied' in means of production, including the commodity money-capital, plus an addition of living labour. The assumption that capitalist households consume, rather than abstain, during replacement is equivalent to the assumption that the commodity money-capital is a means of production with a labour-value that gets transferred to the product.

Definition (12) makes it clear that capitalists consume during replacement. Although the matrix of capitalist consumption coefficients $\mathbf{B}$ is a datum independent of the price system it was nonetheless derived via unit cost prices $\mathbf{m}$. Nonstandard labour-values can be equivalently defined as

$$\begin{aligned} \tilde{\mathbf{v}} &= \tilde{\mathbf{v}}\mathbf{A}^{+}(1 + r_v) \\ \tilde{\mathbf{v}}\bar{\mathbf{w}}^{\mathrm{T}} &= 1, \end{aligned} \tag{18}$$

where $\mathbf{A}^{+}=\mathbf{A}+\bar{\mathbf{w}}^{\mathrm{T}}\mathbf{l}$ is the technique augmented by workers consumption and $r_v=\tilde{\mathbf{v}}\mathbf{c}^{\mathrm{T}}/(\tilde{\mathbf{v}}\mathbf{A}\mathbf{q}^{\mathrm{T}} + \tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}})$ is the labour-value rate of profit (the equivalence of (12) and (18) is proved in the appendix). Definition (18) is independent of the price system.

[8] Alternatively, assume nonstandard labour-values $\tilde{\mathbf{v}}$ and the total labour force L is fixed but the real wage $\mathbf{w}$ is a free variable. Then the wage value equality, $\tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}} = L$, is a hyper-plane equation that represents the real wage possibility frontier. Each point on the surface of the hyper-plane is a possible composition of the real wage $\mathbf{w}$ that may be produced given current technology, capitalist consumption and labour resources. Ratios of nonstandard labour-values, $\omega_{i,j} = \tilde{v}_i/\tilde{v}_j$, represent marginal rates of transformation ('trade-off possibilities') in the real wage between commodities i and j.

6. The labour-cost accounting error

Standard labour-values measure the total direct and indirect labour if capitalists abstain during replacement. In this case the net value equality obtains. Nonstandard labour-values measure total direct and indirect labour if capitalists consume during replacement. In this case the wage value equality obtains.

Both standard and nonstandard labour-values are independent of the price system. Standard labour-values depend on $\mathbf{A}$ and $\mathbf{l}$ and are therefore independent of the real distribution of income. Nonstandard labour-values depend on $\mathbf{A}$, $\mathbf{l}$ and the real wage $\mathbf{w}$ and are therefore dependent on the real distribution of income. In the standard case 'present-day production conditions' refer exclusively to 'technical' features of the economy, whereas in the nonstandard case 'present-day production conditions' also include the 'social' features of the economy, specifically how the net product is divided between workers and capitalists.

For example, consider the standard and nonstandard labour-values for the 2-commodity economy. Standard definition (4), $\mathbf{v} = \mathbf{v}\mathbf{A} + \mathbf{l}$, yields standard labour-values $\mathbf{v} = [\ 36.67 \quad 13.89\ ]$.

The technique augmented by capitalist consumption is

$$\tilde{\mathbf{A}} = \mathbf{A} + \bar{\mathbf{c}}^{\mathrm{T}}\mathbf{m} = \mathbf{A} + \begin{bmatrix} 0.071 & 0.029 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0.571 & 0.229 \\ 0.6 & 0.4 \end{bmatrix}.$$

Hence nonstandard definition (12), $\tilde{\mathbf{v}} = \tilde{\mathbf{v}}\tilde{\mathbf{A}} + \mathrm{l}$, yields nonstandard labour-values $\tilde{\mathbf{v}} = [55.0 \quad 22.68\ ]$.[9]

$\mathbf{v} < \tilde{\mathbf{v}}$ because standard labour-values do not count the labour used-up to produce capitalist consumption goods during replacement. Standard labour values satisfy the net value equality, $\mathbf{vn}^{\mathrm{T}} = 110 = L$. Nonstandard labour-values satisfy the wage value equality, $\tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}} = 110 = L$.

A foundational question in the construction of a labour theory of value is how to measure the replacement costs of commodities by amounts of labour time. Do standard labour-values answer this question?

An input-output matrix describes a network of transformation rates between sectors of an economic system in which commodities are produced 'by means of other commodities' (SRAFFA 1960). The objective 'diffculty of production' (RICARDO [1817] 1996), or 'physical' real cost, of a commodity is implicit in the totality of relations defined by the network structure. Adefinition of labour-value is a *method of reduction* that reduces the 'difficulty of production' of a commodity, implicit in a network of economic relations, to a single, scalar measure of labour time (BIDARD (2004), p. 59).

Systems of measurement define a standard unit in which the measurand is quantified. For example, the 'metre' is the fundamental unit of length. The question, 'How many metres are in one metre?' represents a misunderstanding of the theoretical role of the standard unit because the measure of the standard unit is *by definition* a unit of the standard. In a system of measurement the 'irreducibility' of the standard unit is *a priori*. So the question is similar to querying the colour of a logarithm (MARX [1894] 1971) or the time on the sun (POLLOCK 2004). In a labour theory of value the question, 'What is the labour-value of one unit of direct labour?' is similarly ill-formed: the 'difficulty of production', or real cost, of 1 hour of labour, *measured by labour time*, is 1 hour. No further analysisis possible or required.

[9] Equivalently, nonstandard definition (18), $\tilde{\mathbf{v}}\mathbf{A}^{+} = \lambda\tilde{\mathbf{v}}$, yields characteristic equation $\lambda^2 - 1.082\lambda + 0.142 = 0$. The dominant root is $\lambda_* = 0.929$; hence the labour-value rate of profit $r_\upsilon = 0.076$ or 7.6%. Solving equation $\tilde{\mathbf{v}}(\mathbf{A}^{+} - \lambda_*\mathbf{I}) = \mathbf{0}$ yields $\mathbf{v} = \tilde{\upsilon}_1[1\ 0.412]$. Solving $\mathbf{v}\bar{\mathbf{w}}^{T} = 1$ gives $\tilde{\upsilon}_1 = 55.0$. Hence $\tilde{\mathbf{v}} = [55.0\ 22.68]$.

For example, MARX writes that the expression 'labour-value of labour-power', where labour-power is the capacity to supply labour, denotes the 'diffculty of production' of labour-power; whereas the expression 'labour-value of labour' embodies a confusion: 'the value of labour is only an irrational expression for the value of labour-power'. And further: 'Labour is the substance, and the immanent measure of value, but *has itself no value*.' (MARX ([1887] 1954), p. 503).

The irreducibility of the standard unit manifests in both the standard and nonstandard definitions of labour-value. The labour-value of a commodity is the sum of a series of amounts of direct labour supplied at different stages 'back in time'. At each stage the direct labour supplied *is not further reduced* to its own 'difficulty of production'; that is, both methods of reduction do not reduce direct labour to the real wage and then further reduce the real wage to its vertically integrated labour cost (e.g., see equations (5) and (14)). The theoretical meaning of a unit of labour as a measure of 'difficulty of production' is conceptually independent of the level of the real wage: whether a worker consumed one unit or a thousand units of corn when supplying that hour of labour is irrelevant to the question, 'What is the labour--value of one unit of direct labour?' This property of irreducibility explains why, under the dated interpretation of both standard and nonstandard labour-values, workers abstain from consumption during the hypothetical period of replacement. Since direct labouris not reduced to the real wage it does not enter as a cost of production in the calculation of labour-values. Worker abstention during replacement is therefore a *necessary* property of all methods that aim to reduce the 'difficulty of production' of commodities to labour time.

Capitalism is a 'monetary production economy' in the sense that commodities are produced by means of commodities and the commodity money-capital. In a capitalist economy a unit of corn, iron etc. cannot be produced without advances of money-capital and the simultaneous production of consumption goods for capitalists. Capitalists do not supply money-capital for free. Although it is *possible* that commodities could be produced without money-capital in the 'present-day production conditions' of a capitalist economy *in fact* they are not.[10]

The standard and nonstandard definitions differ in their treatment of the commodity money-capital: the standard definition omits it whereas the nonstandard definition includes it. The omission of money-capital in the standard definition entails that capitalist consumption does not enter as a cost of production in the calculation of labour-values. Hence under the dated interpretation of standard labour--values capitalists *also* abstain from consumption during the process of replacement.

Worker abstention during replacement is a manifestation of the irreducibility of the standard unit in a system of measurement and is therefore a necessary property of any definition of labour-value. In contrast, capitalist abstention during replacementis a *contingent* property of a definition of labour-value. The standard method of reduction, compared to the nonstandard method, is incomplete since the commodity money-capital is not reduced to its labour cost.

Consider that iron is one commodity amongst many produced by an economy. Iron requires a heterogeneous collection of inputs for its production. A method of reduction that does not reduce iron to its labour cost will fail to measure the replacement costs of the economy. In fact, the method will underestimate labour-values since the

[10] In the same way that it is possible that commodities could be produced without iron.

additional 'difficulty of production' incurred by iron production is ignored. Although the commodity money-capital is advanced, rather than produced, the advance uses-up a heterogeneous collection of inputs. A method of reduction that does not reduce money-capital to its labour cost will also fail to measure the replacement costs of the economy. The method will underestimate labour-values since the additional 'difficulty of production' of providing the capitalist class with consumption goods is ignored.

In consequence, standard labour-values do not measure the actual replacement costs of a capitalist economy. They measure the counter-factual replacement costs that would obtain if money-capital were absent.[11] This labour-cost accounting error has been, and continues to be, the major obstacle toward a deeper understanding of the relationship between social labour and monetary phenomena.

7. **The transformation problem**

The classical economists, Smith ([1776] 1994), Ricardo ([1817] 1996) and Marx ([1887] 1954), employed variants of a labour theory of value in order to understand the objective laws that ultimately regulate the prices of reproducible goods. But the existence of 'profits on stock' (Ricardo [1817] 1996) introduced a fundamental theoretical difficulty with this approach.

Prices of production can be reduced to a sequence of wage payments, advanced for different periods, plus the interest received over the duration of the advances. Solve

[11] Nonstandard labour-values equal standard labour-values when profits are zero. Hence standard labour-values are a special case of nonstandard labour--values.

price equation (2) to get $\mathbf{p} = w\mathbf{l}(1+r)(\mathbf{I}-\mathbf{A}(1+r))^{-1}$. Expand the inverse to yield a series representation of prices of production,

(19) $\mathbf{p}=w\mathbf{l}(1+r)+w\mathbf{l}\mathbf{A}(1+r)^2+w\mathbf{l}\mathbf{A}^2(1+r)^3+\cdots+w\mathbf{l}\mathbf{A}^t(1+r)^{n+1}+\ldots$.

Consider that capitalists advance money-capital to fund the process of replacement. For example, at 'date' $t \in [0,-\infty)$ the vector of direct labour supplied is $\mathbf{l}\mathbf{A}^{|t|}$ hence the wage costs advanced to each sector are $w\mathbf{l}\mathbf{A}^{|t|}$. The money-capital advanced to cover wage costs at time t does not return to the capitalist until replacement completes at $t = 0$ when unit outputs are sold. So an advance at t is invested for $|t|+1$ periods of production.

Investments of different duration earn an equal return, or uniform rate of profit, by the application of compound interest (otherwise loans of greater duration earn a lower return). The *final* money value of a portfolio of investments $w\mathbf{l}\mathbf{A}^{|t|}$ that earns uniform interest r for $|t|+1$ periods is $w\mathbf{l}\mathbf{A}^{|t|}(1+r)^{|t|+1}$ by the standard formula for compound interest. Hence each term of series (19) has two cost components, the wages advanced at t, $w\mathbf{l}\mathbf{A}^{|t|}$, plus the interest earnt on the advance for $|t|+1$ periods at rate r, $w\mathbf{l}\mathbf{A}^{|t|}((1+r)^{|t|+1}-1)$.

Labour theories of value imply that prices represent the objective 'difficulty of production' of commodities in terms of the quantities of labour required to produce them. RICARDO suggests that 'every increase of the quantity of labour must augment the value of that commodity on which it is exercised, as every diminuation must lower it' (RICARDO ([1817] 1996), p. 19). It seems natural to propose that commodities with equal labour-values should have identical prices. On this assumption prices of production are proportional to labour-values; that is $v_i = v_j$ if and only if $p_i = p_j$, which implies $p_i/v_i = p_j/v_j = \alpha$, where a is a constant of proportionality.

Write price equation (19) as $\mathbf{p} = w\mathbf{l}\sum_{n=0}^{\infty} \mathrm{A}^n (1+r)^{n+1}$ and standard labour-value equation (5) as $\mathbf{v} = \mathbf{l}\sum_{n=0}^{\infty} \mathrm{A}^n$. Proportionality p = av implies

$$\mathbf{p} - \alpha\mathbf{v} = 0$$

$$w\mathbf{l}\sum_{n=0}^{\infty}\mathbf{A}^n(1+r)^{n+1} - \alpha\mathbf{l}\sum_{n=0}^{\infty}\mathbf{A}^n = 0$$

or

$$\mathbf{l}(w(1+r)-\alpha)+\mathbf{l}\mathbf{A}(w(1+r)^2-\alpha)+\cdots+\mathbf{l}\mathbf{A}^n(w(1+r)^{n+1}-\alpha)+\cdots = 0.$$

Since $\mathbf{l}$ and $\mathbf{A}$ are non-zero α must satisfy

(20) $$\alpha = w(1+r)^{n+1}$$

for *all* $n \in [0,\infty)$. No constant of proportionality α can satisfy this condition in general. Hence prices are not proportional to labour-values.

The problem is that competitive prices include an element of monetary cost that represents the interest earnt over the *duration* of an advance of money-capital. Hence an element of 'time' is introduced into the determination of prices that is unrelated to labour time. So 'time is money' but not in the sense required by a labour theory of value. The total standard labour performed per unit commodity is a simple sum of all the labour applied at each stage, $\mathbf{l}\mathbf{A}^n$. But the price of a commodity is not a simple sum of all the wage costs, $w\mathbf{l}\mathbf{A}^n$. The price includes profit earnt on the wages advanced. The final profit on wages advanced at t, $w\mathbf{l}\mathbf{A}^{|t|}((1+r)^{|t|+1}-1)$, is a function of the rate of profit, r, and the duration of the advance, t. So prices of production depend on the rate of profit that compounds over investment periods. A change in the rate of profit alters prices but leaves labour-values unaltered. So prices can vary due to a cause that is independent of the labour-embodied in commodities. Labour-values cannot in principle fully explain prices if prices can vary independently of labour-values. The presence of 'profits on stock' therefore appears to confound the labour theory of value.

Condition (20) is satisfied if $r = 0$. Smith ([1776] 1994) therefore restricted the applicability of a pure labour theory of value to an 'early and rude state of society' that precedes the 'accumulation of stock'. In this special circumstance the constant of proportionality is the wagerate, $\alpha = w$, and the labour-embodied in a commodity, v_i, equals the labour-commanded by a commodity, p_i/w.

Ricardo concluded that the principle that labour-embodied determines the price of commodities must be 'considerably modified' and subject to exceptions, yet nonetheless remains the 'foundation of all value'. He noted that if the ratio of dead labour (in the form of means of production) to living labour is identical in all industries then prices are proportional to labour values (Ricardo ([1817] 1996), p. 31). Following Marx ([1894] 1971) call this ratio the 'organic composition of capital' and define it as $k_i = \mathbf{v}\mathbf{A}^{(i)}/\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}} l_i$ for sector i, where $\mathbf{A}^{(i)}$ is the ith column of $\mathbf{A}$. Uniform organic compositions obtain when $k_i = \mathrm{k}_j = k$ for all i and j; that is

$$\mathbf{v}\mathbf{A} = k\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}}\mathbf{l}. \tag{21}$$

Hence $\mathbf{l} = (1/k\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}})\mathbf{v}\mathbf{A}$. Substitute into standard labour--value equation (4) to get $\mathbf{v} = (\mathbf{v}\mathbf{A} + (k\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}})\mathbf{v}\mathbf{A})/(k\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}})$. Substitute for $\mathbf{v}\mathbf{A}$ using (21) to get $\mathbf{v} = k\mathbf{v}(\mathbf{A} + \bar{\mathbf{w}}^{\mathrm{T}}\mathbf{l})$. In this special case standard labour-values are

$$\mathrm{v}_k = k\mathbf{v}_k\mathbf{A}^{+}. \tag{22}$$

By comparison with price equation (10) it follows that $k = 1/\lambda_* = r+1$. So $\mathbf{v}_k = (\mathbf{v}_k\bar{\mathbf{w}}^{\mathrm{T}})\mathbf{l}\sum_{t=0}^{\infty}\mathbf{A}^t(1+r)^{t+1}$ and condition (20) is satisfied with $\alpha = w/(\mathbf{v}_k\bar{\mathbf{w}}^{\mathrm{T}})$, confirming RICARDO'S claim that prices are proportional to labour values in this special case.

Industries with identical organic compositions have profits that compound proportionally to embodied labour during replacement. In conditions of a uniform rate of profit the labour theory of valueis approximately correct to the extent that the organic compositions of capital are close to uniform. But in general organic compositions of capital are not uniform.

RICARDO acknowledged the existence of contradictions in his theory of value. 'I cannot get over the dfficulty of the wine which is kept in the cellar for three or four years ... which perhaps originally had not 2 s. expended on it in the way of labour, and yet comes tobe worth £100' (HOLLANDER 1896). Since by definition no additional labour is applied during fermentation the increase in value appears to be compensation for the *time* that the initial investment of 2 s. is 'locked up' in the form of wine.[12]

MARX ([1887] 1954) assumed price-value proportionality in Volume I of *Capital*. On this basis total profitis the monetary representation of the total unpaid labour of the working class, or surplus-value. But to maintain this critique Marx had to resolve the contradiction between the pure labour theory of value and capitalist prices. MARX ([1894] 1971) turned to the problem in his unfinished notes published as Volume III of *Capital*.

In MARX's theory the labour value, v_i, of a commodity consists of three components: constant capital, $C_i = \mathbf{v}\mathbf{A}^{(i)}$,

[12] An observation that motivates the idea that profitis 'only ajust compensation for the time that profits were withheld' (RICARDO [1817] 1996).

which is means of production used-up, variable capital, $V_i = \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}} l_i$, which is workers' wages, and surplus-value, S_i, which is unpaid labour-time; that is $v_i = C_i + V_i + S_i$. But only living labour *creates* surplus-value. Sotheamount of surplus-value producedby each sector (or 'sphere of production') depends on the variable, not the constant, capital.

MARX initially assumes that the rates of surplus value, or degrees of exploitation, are equal, that is $S_i/V_i = S_j/V_j = e$ for all i and j, where $e = \mathbf{v}\mathbf{c}^{\mathrm{T}}/\mathbf{v}\mathbf{w}^{\mathrm{T}}$. If prices are proportional to labour values the rates of profit in each sector are

$$r_i = \frac{S_i}{C_i + V_i} = \frac{S_i}{V_i}\frac{1}{\frac{C_i}{V_i}+1} = e\frac{1}{\frac{C_i}{V_i}+1},$$

which are equal only if the organic composition of capitals, that is the ratios $C_i/V_i = C_j/V_j$ are equal, for all i and j. Hence, 'in the different spheres of production with the same degree of exploitation, we find considerably different rates of profit corresponding to the different organic composition of these capitals' (MARX ([1894] 1971), p. 155).

MARX proposes that capitalist prices are *transformed* labour-values that *redistribute* the surplus-value created in each productive sector. 'The rates of profit prevailing in the various branches of production are originally very different' (MARX ([1894] 1971), p. 158) but the different rates 'are equalised by competition to a single general rate of profit' (MARX ([1894] 1971), p. 158). At which point, 'although in selling their commodities the capitalists of various spheres of production recover the value of the capital consumed in their production, they do not secure the surplus-value, and consequently the profit, created in their own sphere by the production of these commodities. What they secure is only asmuch surplus-value, andhence profit, as falls, when uniformly distributed, to the share of every aliquot part of the total social capital from the total social surplus-value, or

profit, produced in a given time by the social capital in all spheres of production' (MARX ([1894] 1971), p. 158). So capitalists share the available pool of surplus-value in proportion to the size of the money-capitals they advance rather than the size of the workforces they employ. For example, the money-capital 'locked up' in the form of wine increases in value because the investing capitalist gets a share of the surplus-labour performed in *other* spheres of production during the process of fermentation.

MARX computes a general rate of profit by dividing the total surplus-value by the total value of the constant and variable capital,

$$r_v = \frac{\sum_{i=1}^{n} S_i q_i}{\sum_{i=1}^{n} (C_i + V_i) q_i} = \frac{S}{C + V}, \tag{23}$$

where $S = \mathbf{v}\mathbf{c}^{\mathrm{T}}$ is the total surplus-value, $C = \mathbf{v}\mathbf{A}\mathbf{q}^{\mathrm{T}}$ is the total constant capital and $V = \mathbf{v}\mathbf{w}^{\mathrm{T}}$ is the total variable capital.[13] According to MARX, the 'prices of production' that effect the redistribution of surplus-value are

$$p_i^{\star} = \alpha[C_i + V_i + r_v(C_i + V_i)] \tag{24}$$

for all i, where α is a constant of proportionality. Define $k_i^{\star} = \alpha(C_i + V_i)$ as the 'cost-price' of commodity i. 'Hence, the price of production of a commodity is equal to its cost--price plus the profit, alloted to it in per cent, in accordance with the general rate of profit, or, in other words, to its cost-price plus the average profit' (MARX ([1894] 1971), p. 157); that is, $p_i^{\star} = k_i^{\star} + k_i^{\star} r_v$.

MARX'S 'prices of production' are not proportional to labour-values; in general $p_i^{\star} \neq \alpha v_i$. 'One portion of the

[13] Derive equality (23) by $\sum_{i=1}^{n} S_i q_i / \sum_{i=1}^{n} (C_i + V_i) q_i = e \sum_{i=1}^{n} V_i q_i / \mathbf{v}\mathbf{A}^{(i)} q_i + \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}} l_i q_i = e \sum_{i=1}^{n} \mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}} l_i q_i / (\mathbf{v}\mathbf{A}\mathbf{q}^{\mathrm{T}} + \mathbf{v}\mathbf{w}^{\mathrm{T}}) = \mathbf{v}\mathbf{c}^{\mathrm{T}} / (\mathbf{v}\mathbf{A}\mathbf{q}^{\mathrm{T}} + \mathbf{v}\mathbf{w}^{\mathrm{T}})$.

commodities is sold above its value in the same proportion in which the other is sold below it. And it is only the sale of the commodities at such prices that enables the rate of profit for capitals [to be uniform], regardless of their different organic composition' (MARX ([1894] 1971), p. 157). In MARX's view the divergence of prices from labour-values is not an exception to the labour theory of value but a necessary mechanism of the redistribution of surplus-value. Nonetheless the labour theory of value continues to hold in the aggregate because the transformation is *conservative*: the redistribution of surplus-value neither creates or destroys the labour embodied in commodities. So MARX claimed that three aggregate equalities are invariant over the transformation: (i) the rate of profit is equal to the ratio of total surplus-value to the total labour-value of capital advanced; (ii) 'the sum of the profits in all spheres of production must equal the sum of the surplus-values', (MARX ([1894] 1971), p. 173); and (iii) 'the sum of the prices of production of the total social product equal the sum of its value' (MARX ([1894] 1971), p. 173).[14]

MARX's 'prices of production' are computed from the assumption that the price and value rates of profit are equal; hence equality (i) is true by definition. Also,

(ii) total profit is proportional to the total surplus-value,

$$\begin{aligned} r_v \sum_{i=1}^{n} k_i^{\star} q_i &\propto r_v \sum_{i=1}^{n} (C_i + V_i) q_i \\ &= S \\ &= e \sum_{i=1}^{n} V_i \end{aligned}$$

[14] Marx assumes a unit constant of proportionality between labour-values and prices when formulating his conservation rules.

and (iii) total price is proportional to total value,

$$\begin{aligned} \sum_{i=1}^{n} p_i^{\star} q_i &\propto \sum_{i=1}^{n} [C_i + V_i + r_v(C_i + V_i)]\, q_i \\ &= C + V + S \\ &= \sum_{i=1}^{n} v_i, \end{aligned}$$

confirming MARX's claim that labour-value is conserved in price, despite the divergence of prices from labour-values due to 'profits on stock'. The classical contradiction appears solved.

But MARX immediately critiques his own derivation. He observes that, 'we had originally assumed that the cost-price of a commodity equalled the *value* of the commodities consumed in its production. But for the buyer the price of production of a specific commodity is its cost-price, and may thus pass as a cost-price into the prices of other commodities. Since the price of production may differ from the value of a commodity, it follows that the cost-price of a commodity containing the price of production of another commodity may also stand above or below that portion of its total value derived from the value of the means of production consumed by it. It is necessary to remember this modified significance of the cost-price, and to bear in mind that there is always the possibility of an error if the cost--price of a commodity in any particular sphere is identified with the value of the means of production consumed by it. Our present analysis does not necessitate a closer examination of this point' (MARX ([1894] 1971), p. 165). The transformation procedure, like the whole of Volume III of *Capital*, is unfinished.

The problem is that MARX's 'prices of production', defined by equation (24), are calculated on the basis of *untransformed* cost-prices, $k_i^\star = \alpha(C_i + V_i)$, which are *proportional* to labour-value. LIPPI (1979) remarks that MARX knew that 'the magnitudes on the basis of which surplus-value has been redistributed – that is, capital advanced, measured in value – are not identical to the prices at which elements of capital are bought on the market. He therefore admits that the prices previously calculated must be adjusted' (LIPPI 1979). Market prices in conditions of self-replacing equilibrium and a uniform rate of profit are defined by price equation (2) not MARX's equation (24). The transformation problem is then the logical impossibility of MARX's conservation claims once this adjustment is made.

To see this write MARX's aggregate equalities using price equation (2): (i) the rate of profit equals the labour value rate of profit ($r=\mathbf{pc}^T/(\mathbf{pAq}^T+\mathbf{pw}^T)=\mathbf{vc}^T/(\mathbf{vAq}^T+\mathbf{vw}^T)=r_v$); (ii) total profitis proportional to surplus-value ($\mathbf{pc}^T \propto \mathbf{vc}^T$); and (iii) total price is proportional to total value ($\mathbf{pq}^T \propto \mathbf{vq}^T$). Following ABRAHAM-FROIS and BERREBI (1997) assume claim (iii) holds such that $\mathbf{pq}^T = \alpha\mathbf{vq}^T$. (ii) can be written $\mathbf{pq}^T - \mathbf{pA}^+\mathbf{q} = \alpha(\mathbf{vq}^T - \mathbf{vA}^+\mathbf{q}^T)$. Replacing $\mathbf{pq}^T$ by $\alpha\mathbf{vq}$ gives $\mathbf{pA}^+\mathbf{q}^T = \alpha\mathbf{vA}^+\mathbf{q}^T$. But $\mathbf{pA}^+ = (1/(1+r)\mathbf{p}$. Hence $\mathbf{vq}^T = \mathbf{vA}^+(1+r)\mathbf{q}^T$, or equivalently,

$$\mathbf{vx}^T = 0, \tag{25}$$

where $\mathbf{x} = (\mathbf{I} - \mathbf{A}^+(1+r))\mathbf{q}^T$. The set of cases in which MARX's conservation claims hold are defined by condition (25), the orthogonality of vectors $\mathbf{v}$ and $\mathbf{x}$. If any one claim is assumed to hold then, unless condition (25) is satisfied, at least one of the remaining two claims is false. The orthogonality conditionis satisfied in some special cases, such as zero profits or uniform organic compositions of capital. But in general (25) does not hold and there is no economic reason why it should. Hence prices of production are not conservative transforms of values and MARX's proposed solution to the classical contradiction fails. Since 'what does not hold in the special case cannot claim general validity' (VON BORTKIEWICZ 1975) price cannot measure labour-value and 'there is no rigorous quantitative connection between the labour time accounts arising from embodied labour coefficients and the phenomenal world of moneyprice accounts' (FOLEY 2000).

For example, consider the relations between prices and standard labour-values for the 2-commodity economy. Prices of production are $\mathbf{p} = p_1[1\ 0.412\ ]$. Standard labour values are $\mathbf{v} = [\ 36.67\ 13.89\ ]$. There is no a such that $\mathbf{p} = \alpha\mathbf{v}$; hence prices are not proportional to labour-values. The value rate of profit $r_v = S/(C + V)$ =7.82 or 7.8%, which is not equal to the actual rate of profit r =7.6%. If aggregate price is proportional to aggregate labour-value then $\alpha_1 = \mathbf{pq}^T/\mathbf{vq}^T = 0.028p_1$. If total profit is proportional to surplus-value then $\alpha_2 = \mathbf{pc}^T/\mathbf{vc}^T = 0.027p_1$. The constants of proportionality are inconsistent, $\alpha_1 \neq \alpha_2$. MARX's conservation claims do not hold.

The transformation problem is the primary reason for the modern rejection of the logical tenability of a labour theory of value. The debate has generated a large literature spanning over one hundred years. STEEDMAN (1981) provides the definitive statement of the negative consequences of the transformation problem for MARX's value theory. According to STEEDMAN, MARX's value theory must be rejected on two grounds. First, the theory is *internally inconsistent* because MARX 'assumes that $S/(C + V)$ is the rate of profit but then derives the result that prices diverge from values, which means precisely, in general, that $S/(C + V)$ is notthe rate of profit' (STEEDMAN (1981), p. 31). Second, the theory is *redundant* because 'profits and prices *cannot* be derived from the ordinary value schema, that $S/(C + V)$ is *not* the rate of profit and that total profit is *not* equal to surplus value' (STEEDMAN (1981), p. 48). STEEDMAN notes, following SAMUELSON (1971), that given a technique and a real wage (the 'physical schema') one can determine (a) profits and prices and (b) labour-values. But due to the non-satisfaction of condition (25) there is in general 'no way' of relating (a) and (b). Despite MARX's efforts a theory of value based exclusively on labour-cost *cannot account* for price phenomena.

8. Dissolution of the transformation problem

The theoretical contradiction between the labour theory of value and 'profits on stock' begins at the birth of political economy. The profit component of price seems unrelated to the labour embodied in a commodity. Marx is unique in proposing that a general rate of profit is a mechanism for the redistribution of surplus-labour between members of the capitalist class. The profit of an individual capitalist is not the surplus-labour produced in the sector they fund but a share of the total surplus-labour produced in the economy as a whole. The distributional rule of equal returns to money-capital advanced obscures the origin of profit in surplus-labour. Marx seeks therefore to not only resolve the contradiction but also explain the necessity of its appearance in economic theory. But this line of thought is interrupted by the transformation problem.

Money-capital has a price, the rate of profit, but also an associated labour cost, the direct and indirect labour used-up to produce capitalist consumption goods per unit of money-capital advanced. Prices of production count the price of money-capital as a monetary cost of production. But standard labour-values do not count the labour-cost of money-capital as a real cost of production. The price of money-capital refers to labours that are not counted; hence there cannot be

a conservative transform between standard labour-values and prices. The asymmetrical treatment of thecommodity money-capital – present as a monetary cost in the price system but absent as a labour cost in the value system – is the cause of the transformation problem.

Since money-capital is presentas a labour cost in the nonstandard value system the asymmetry between the dual accounting systems is removed. In consequence, prices of production are proportional to nonstandard labour-values.

To see this substitute for $r = \mathbf{p}\bar{\mathbf{c}}^{\mathrm{T}}$ in price equation (2) to get $\mathbf{p} = (\mathbf{pA} + \mathbf{l}w) + (\mathbf{pA} + \mathrm{l}w)\mathbf{p}\bar{\mathbf{c}}^{\mathrm{T}}$. Since cost prices $\mathbf{m} = \mathbf{pA} + \mathrm{l}w$ then $\mathbf{p} = \mathbf{pA} + \mathbf{p}\bar{\mathbf{c}}^{\mathrm{T}}\mathrm{m} + \mathrm{l}w$. An equivalent expression for prices of production is therefore

$$\mathbf{p} = \mathbf{p}\tilde{\mathbf{A}} + \mathbf{l}w.$$

Nonstandard labour-values are

$$\tilde{\mathbf{v}} = \tilde{\mathbf{v}}\tilde{\mathbf{A}} + \mathbf{l}.$$

Hence prices and nonstandard-labour values are related by

$$\mathbf{p} = w\tilde{\mathbf{v}}, \tag{26}$$

where the money wage, w, is the constant of proportionality. The intimate connection between price and labour-cost, a fundamental intuition of the classical labour theory of value, is restored.

For example, consider the relations between prices and nonstandard labour-values for the 2-commodity economy. Prices of production are $\mathbf{p} = p_1[1 \;\; 0.412]$. Nonstandard labour values are $\tilde{\mathbf{v}} = [55.0 \;\; 22.68]$. Prices and nonstandard labour values are proportional, $\mathbf{p} = \alpha\tilde{\mathbf{v}}$ with constant of proportionality $\alpha = 0.018p_1 = \mathbf{pw}^{\mathrm{T}}/L = w$.

Nonstandard labour-values are therefore free of the classical contradictions of the labour theory of value due to the inclusion of the labour-value of money-capital.

For example, the nonstandard labour theory of value is not restricted to pre-civilised times. Standard labour-values are a special case of nonstandard labour-values when $r = 0$. Prices are proportional to nonstandard labour-values regardless of the presence or absence of 'profits on stock'. In all states of self-replacing equilibrium the labour-embodiedin a commodityis equal to thelabourcommanded by the commodity.

For example, the nonstandard labour theory of value is not restricted to circumstances in which the organic compositions of capital are uniform. De?ne the nonstandard organic compositions of capital as $\tilde{k}_i = \tilde{\mathbf{v}}\mathbf{A}^{(i)}/\tilde{\mathbf{v}}\bar{\mathbf{w}}^{\mathrm{T}}l_i$ for all i. Prices are proportional to nonstandard labour-values regardless of the distribution of nonstandard organic compositions. In the special case of uniform standard organic compositions of capital, $k_i = k_j = k$ for all i and j, standard labour-values are proportional to nonstandard labour-values, $\mathbf{v}_k = (\mathbf{v}_k\bar{\mathbf{w}}^{\mathrm{T}})\tilde{\mathbf{v}}$. In this case the omission of money-capital from the standard value system, and the associated labour it represents, accidentally has a proportionate effect.

For example, a transformation from nonstandard labour-values to prices of production is not required. Prices are proportional to nonstandard labour-values even in the case of uniform profits. Hence, in the nonstandard value system, all MARX'S expectations regarding the conservation of labour-time in price are met: (i)the rate of profit equals the labour-value rate of profit,

$$r = \frac{\mathbf{p}\mathbf{c}^{\mathrm{T}}}{\mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}} + \mathbf{p}\mathbf{w}^{\mathrm{T}}} = \frac{\tilde{\mathbf{v}}\mathbf{c}^{\mathrm{T}}}{\tilde{\mathbf{v}}\mathbf{A}\mathbf{q}^{\mathrm{T}} + \tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}}} = \frac{S}{C+V} = r_v, \tag{27}$$

(ii) profit is proportional to surplus-value,

$$\mathbf{p}\mathbf{c}^{\mathrm{T}} = w\tilde{\mathbf{v}}\mathbf{c}^{\mathrm{T}}\, wS,$$

and (iii) total price is proportional to total value,

$$\mathbf{p}\mathbf{q}^{\mathrm{T}} = w\tilde{\mathbf{v}}\mathbf{q}^{\mathrm{T}} = w(C + V + S).$$

Total profit is therefore the monetary representation of the total unpaid labour of the working class. In consequence, the standard criticisms of the labour theory of value do not apply: nonstandard labour-values are not internally inconsistent, since $r = r_v$, nor redundant, since prices may be derived from the value system by scaling by the money wage w. A theory of value based exclusively on labour-cost *can account* for price phenomena: labour-values and prices are 'two sides of the same coin'.

8.1. Remarks on the origin of surplus-value. Marx calls the capacity to work 'labour-power'. Capitalists buy labour-power in the labour market. The labour embodied in the real wageis the labour-value of labour-power. But during production the exercise of labour-power adds a variable quantity of new labour to the product that exceeds the labour-embodied in the real wage ('the value of labour must always be less than the value it produces' (MARX [1887] 1954), p. 505, emphasis added). The *origin* of surplus-value, and therefore profit, is the use-value of labour-power, a commodity that is '*a source not only of value, but of more value than it has itself*' (MARX ([1887] 1954), p. 188). Labour is therefore 'the universal value-creating element' (MARX ([1887] 1954), p. 506) with the unique causal power to break the symmetry between cost and revenue in order to generate a surplus. Constant capital, consisting of material inputs, machinery and tools etc., is merely a passive instrument during this process.

In the special case of self-replacing equilibrium the technique and labour coefficients are fixed. By definition nominal firm costs, including the interest paid on borrowed money-capital, are equal to nominal firm revenues. Profit exists although the symmetry of the price system is unbroken.

A natural viewpoint (e.g., SHAIKH (1987)) is that standard labour-values quantify Marx's explanation of the origin of surplus-value since the standard labour-value of the real wageis *less* than the length of the working day, thatis $\mathbf{v}\mathbf{w}^{T} < L$. The replacement cost of labour does not equal the value it adds since 1 unit of direct labour generates a surplus-value of $1 - \mathbf{v}\bar{\mathbf{w}}^{T}$ units. In accordance with MARX's explanation the symmetry of the standard value system is broken.

In contrast, the nonstandard labour-value of the real wage *equals* thelength of the workingday, thatis $\tilde{\mathbf{v}}\mathbf{w}^{T} = L$. The replacement cost of labour equals the value it adds. Nonetheless, 1 unit of direct labour generates a surplus-value of $\tilde{e}$ units. Contra MARX, surplus-value exists although the symmetry of the nonstandard value system is unbroken.

In both cases surplus-value is the difference between the value of the net product workers create and the value of the wage goods they receive. In the standard approach $\mathbf{v}\mathbf{n}^{T} = L > \mathbf{v}\mathbf{w}^{T}$, whereas in the nonstandard approach $\tilde{\mathbf{v}}\mathbf{w}^{T} = L(1+ \tilde{e}) > \tilde{\mathbf{v}}\mathbf{w}^{T}$. The approaches differ over how the net product is valued. In the standard approach L units of labour are required to replace the net product since capitalists abstain during replacement. In the nonstandard approach $L(1+ \tilde{e})$ units of labour are required to replace the net product since capitalists consume during replacement. The difference is due to the treatment of money-capital. In the standard approach direct labour transfers the labour-embodied in means of production and adds value in excess of its replacement cost. In the nonstandard approach direct

labour transfers the labour-embodied in means of production, *including the commodity money-capital*, and adds value equal to its replacement cost.

MARX devotes considerable attention to money-capital in volumes II and III of *Capital*. Money-capital has a use-value to fund production (he calls it the 'prime motor' of capitalist production). Money-capital also has an exchange-value ('it is not until capital is money-capital that it becomes a commodity, whose capacity for self-expansion has a definite price quoted every time in every prevailing rate of interest' (MARX [1894] 1971). Labour may be embodied in any kind of use-value: 'Value is independent of the particular use-value by which it is borne, but it must be embodied in a use-value of some kind' ((MARX [1887] 1954), p. 183); and although 'we leave out of consideration its purely symbolical representation by tokens' ((MARX [1887] 1954), p. 196) for the embodiment of labour 'it is a matter of complete indifference what particular object serves this purpose' ((MARX [1887] 1954), p. 196). Money-capital is money that commands a price. The *quantity* of money-capital advanced to firms is a 'purely symbolical representation' of the cost of goods that may be bought with it; in contrast, the *total price* of the advanced money-capital, received by capitalist households, represents the cost of money-capital itself. In conditions of simple reproduction this cost is the direct and indirect labour required to produce capitalist consumption goods. Yet money-capitalis the only commodity in MARX's theory with a use-value and an exchange-value but not a labour-value.

The reason for MARX's anomalous treatment of money-capital is in part due to his out-of-equilibrium, symmetry-breaking explanation of the origin of surplus-value. In MARX's view surplus-value lacks a corresponding labour cost that could in principle be transferred to the product:

'The characteristic feature of variable capital is that a definite, given (and as such constant) part of capital, a given sum of values ... is exchanged for a self-expanding, value-creating power, viz., labour-power, which not only reproduces its value, paid by the capitalist, but simultaneously produces a surplus-value, *a value not existing previously and not paid for by any equivalent*' ((MARX [1893] 1974), p. 221-222, emphasis added).

MARX did not formulate his theory in the precise but restricted framework of general equilibrium. The nonstandard approach suggests that the *origin* of new surplus-value in the difference between the value of labour-power and the value it adds is essentially an out-of-equilibrium event that should be distinguished from the *reproduction* or transfer of existing surplus-value. In contrast to the standard approach, the 'two-fold nature' of labour that 'at one and the same time, it must in one character create value, andin another character preserve or transfer value' ((MARX [1887] 1954), p. 193) is a contingent rather than a necessary property of labour-power: whether labour creates value in production depends on how it acts. In an idealised state of equilibrium the unique causal power of human labour to alter the conditions of production is absent: labour merely constantly transfers value without variation. In particular, surplus-value is already embodied in the form of capitalist consumption goods that form part of the reproduction costs, both nominal and real, of the 'present-day production conditions' of the economy. In this special-case the origin of surplus-value in a prior broken symmetry between the nonstandard value of labour-power and the value it adds has been effaced.[15]

[15] Or symmetry-preserving generalisations such as proportionate growth. See the appendix for a nonstandard analysis of this case.

8.2. Remarks on the transformation. The nonstandard approach entails we reject the classical view that prices diverge from labour-values due to profit-equalizing prices of production. Yet the approach is consistent with Marx's view that the contradiction between the law of value and the law of uniform profits is resolved by the redistribution of surplus-value during the formation of a uniform rate of profit. To fully pursue this issue requires a dynamic model of the classical process of 'gravitation' so I will make only brief remarks here.

MARX's transformation procedure is both a logical device and a sketch of a real economic process. In chapter 9 of Volume III of *Capital*, MARX assumes a uniform rate of exploitation and distinguishes the surplus-value *produced by workers* in a given 'sphere of production', $S_i^W = eV_i$, from the surplus-value realised by the individual capitalist in a given 'sphere of production', $S_i^C = (C_i + V_i)r_v$.

The transformation starts with unequal rates of pro?t and $S_i^W = S_i^C$ for all *i*. A uniform rate of profit forms due to capital reallocation from low to high profit sectors. During this process the surplus-value gets redistributed between individual capitalists. The transformation stops with equal rates of profit and $S_i^W \neq S_i^C$. This terminal state is hypothetical since the continual creation of new surplus-value by labour contradicts the distributional rule of equal returns to money-capital invested: the 'gravitational' tendency toward profit-equalising prices of production is not fully realised.

We can compare the start and end states of MARX's proposed transformation. Assume all individual capitalists consume the same consumption bundle. In the standard value system the local surplus-value produced by workers is, $S_i^W = eV_i = \mathbf{vc}^{\mathrm{T}}(l_i/L)$, whereas the local surplus-value realised is the labour embodied in the commodities consumed by the individual capitalist, $S_i^C = \mathbf{v}\bar{\mathbf{c}}^{\mathrm{T}} m_i'$, where each m_i' is an arbitrary unit cost price that corresponds to any distribution of profit rates across sectors.

MARX's start state, $S_i^W = S_i^C$ for all i, implies that unit cost prices are proportional to the direct labour expended in the given sector,[16]

$$m_i' = \frac{M}{L} l_i, \tag{28}$$

where M/L is the ratio of total money-capital advanced to total direct labour. In this case nonstandard labour-values simplify to[17]

$$\tilde{\mathbf{v}} = \tilde{\mathbf{v}}\mathbf{A} + \mathbf{l}(\frac{M}{L}\tilde{\mathbf{v}}\bar{\mathbf{c}}^{\mathrm{T}} + 1), \tag{29}$$

Hence, in the special case when (standard) surplus-value produced equals (standard) surplus-value realised, nonstandard labour-values are proportional to standard labour--values, $\tilde{\mathbf{v}} = k\mathbf{l}(\mathbf{I} - \mathbf{A})^{-1} = k\mathbf{v}$, where k is the constant of proportionality.

In MARX's end state, $S_i^W \neq S_i^C$, in which surplus-value produced does not equal surplus-value realised, nonstandard labour-values are not proportional to standard labour-values. But the aggregate sum of (nonstandard) surplus-value produced by workers equals the total (nonstandard) labour--value realised by capitalists,[18]

$$L\tilde{e} = \tilde{\mathbf{v}}\bar{\mathbf{c}}^{\mathrm{T}} \sum_{i=1}^{n} m_i q_i, \tag{30}$$

where $\tilde{e} = \tilde{\mathbf{v}}\mathbf{c}^{\mathrm{T}} / \tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}}$ is the nonstandard rate of exploitation. So the labour-value realised by an individual capitalist,

[16] $\mathbf{v}\mathbf{c}^{\mathrm{T}}(l_i/L) = \mathbf{v}\bar{\mathbf{c}}^{\mathrm{T}} m_i' \Rightarrow l_i/L = m_i'/M$.

[17] Substitute (28) into (12) to get $\tilde{\mathbf{v}} = \tilde{\mathbf{v}}\mathbf{A} + \tilde{\mathbf{v}}\bar{\mathbf{c}}^{\mathrm{T}}\mathbf{m}' + \mathbf{l} \Rightarrow \tilde{\mathbf{v}}\mathbf{A} + \frac{M}{L}\tilde{\mathbf{v}}\bar{\mathbf{c}}^{\mathrm{T}}\mathbf{l} + \mathbf{l}$.

[18] $\tilde{\mathbf{v}}\bar{\mathbf{c}}^{\mathrm{T}} \sum_{i=1}^{n} m_i q_i = \tilde{\mathbf{v}}\mathbf{c}^{\mathrm{T}} = L(\tilde{\mathbf{v}}\mathbf{c}^{\mathrm{T}}/\tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}}) = L\tilde{e}$.

$\tilde{\mathbf{v}}\bar{\mathbf{c}}^{\mathrm{T}} m_i q_i$, is a share, proportionate to the money-capital they advance, of the total surplus-value produced in all 'spheres of production', $L\tilde{e}$. In the nonstandard approach, therefore, capitals receive a share of the surplus-labour performed in *other* spheres of production due to the existence of equal returns to money-capital, in accordance with MARX's view.[19]

8.3. Summary. The classical authors believed that prices diverge from labour costs due to 'profits on stock'.

[19] Shaikh (1984) explains thatthe non-conservation of labour-value in price is due to the transfer of labour-value out of the 'circuit of capital'and into capitalists' 'circuit of revenue' where profits are spent on consumption goods. Shaikh's explanation is an implicit and partial recognition of the absence of the labour-value of money-capital in the standard value system. Shaikh employs the standard definition of labour-value and therefore follows Ricardo in viewing the relationship between price and labour-value as approximate *in theory*, although he argues that individual deviations of prices from labour-values are 'quite moderate'. He initiates an important empirical research programme to measure the size of the deviations (e.g., Shaikh and Tonak (1994), Ochoa (1988), Cockshott et al. (1995), Cockshott and Cottrell (1997), Zachariah (2006)). Notable resultsare (i) standard labour-values, in contrast to other real costbases, such as oil, are closely correlated with market prices, (ii) there is no evidence that a uniform rate of profit is realised (Zachariah 2006, Wells 2006), and (iii) a significant *negative* correlation obtains between profit rates and empirical analogues of the standard organic composition of capital (Cockshott and Cottrell 2003, Zachariah 2006). In sum the empirical data indicates that standard labour-values are attractors for market prices. Standard labour-values are also a special case of nonstandard labour-values when in each sector the surplus-value produced equals the surplus-value realised. The empirical results may therefore indicate that Marx's pre-transformation conditions are reasonable approximations when studying an aggregated snapshot of a capitalist economy in which the continual production of new surplus-value subverts the tendency toward the formation of a general rate of profit. Under such conditions standard and nonstandard labour-values are convergent and make similar predictions. But these empirical studies are tangential to the problem that the non-conservation of labour-value in price implies that the labour theory of value *in theory* cannot fully explain competitive prices and therefore is only an approximation. The existence of an empirical correlation between market prices and standard labour-values is consistent with there being another 'less powerful cause' (Ricardo (2005), p. 404-405), other than labour time, of the variation in relative prices.

This premise has been universally accepted. But it is false. In general, Marxian authors have maintained divergence and aggregate conservation of labour-value in price, whereas critics have maintained divergence but denied aggregate conservation of labour-value in price. But both sides of the argument are mistaken: there is no divergence and there is aggregate conservation. The transformation problem is the inverted appearance of an accounting error due to the omission of the labour-value of money-capital. Nonstandard labour-values, not standard labour-values, are the labour costs that competitive prices represent.

9. **The problem of an invariable measure of value**

Theories of economic value try to identify price-independent parameters that are the ultimate source or cause of price (COHEN 1993). RICARDO aimed to explain the laws that regulate the natural prices of reproducible commodities in terms of their 'difficulty of production' measured in labour time. Natural prices, or prices of production, are the stable prices that are robust to 'accidental and temporary deviations' between supply and demand (RICARDO ([1817] 1996), Ch. 5). And reproducible commodities are those 'that may be multiplied ... almost without any assignable limit, if we are disposed to bestow the labour necessary to obtain them' (RICARDO ([1817] 1996), p. 18).

Consider a plot of land A that is twice the area of a plot of land B. But at a later date plot A is three times the area of plot B. Does this *relative* change imply that plot A has increased in size, plot B has decreased in size, or some combination of these causes? We need an *absolute* measure of size that is *invariable* over time to answer this question. The 'metre' is such an invariable standard. We measure the

absolute sizes of plot A and B in metres, both before and after the change, in order to determine the cause of the variation in relative size.

Although we now take for granted such standards of measure, the definition and adoption of the metre in 1793 by post-revolutionary France was accompanied by much theoretical debate and reflection (RONCAGLIA 2005). RICARDO, a contemporary of these events, applies the same kind of reasoning to the problem of economic value.

Any system of prices is relative since an increase in the amount of currency that circulates commodities produces a general increase of all prices. RICARDO considers an example of game that exchanges for two salmon. Then at a later date the game exchanges for three salmon. RICARDO asks: does this change in relative price imply that game is now more difficult to produce, salmon easier to produce, or some combination of these causes? (RICARDO ([1817] 1996), p. 27-29). To answer this question RICARDO demands an absolute measure of economic value that is invariable over time.

> When commodities variedin relative value [natural price] it would be desirable to have the means of ascertaining which of them fell and which rose in real value, and this could be effected only by comparing them one after another with some invariable standard measure of value, which should itself be subject to none of the fluctuations to which other commodities are exposed. (RICARDO ([1817] 1996), p. 38).

In RICARDO's thought the problem of a measure of value and the aim of elucidating the laws that regulate natural prices are closely identified (SRAFFA (2005), p. xli). An invariable standard is an objective measuring rod, independent of the price system, which measures 'difficulty of production'. Armed with an invariable standard we can identify which commodity has undergone a change in its conditions of production that is the ultimate cause of the observed changein relative prices. A public standard of economic value is needed otherwise the theory of economic value collapses into subjectivity, leaving 'every one to chuse his own measure of value' (RICARDO (2005), p. 370). RICARDO's search for an invariable standard is therefore motivated by the requirement to objectively verify his theory of natural prices (COLLIOT-THÉLÈNE 1979, BIDARD 2004).

In certain special cases the relationship between an invariable standard and natural prices is transparent. For example, in circumstances of equal organic compositions of capital the relative prices of commodities are 'entirely regulated by the quantity of labour realized' (RICARDO ([1817] 1996), p. 27) in them. In consequence,

> If with the same quantity of labour a less quantity of fish or a greater quantity of game were obtained, the value [natural price] of fish would rise in comparison with that of game. If, on the contrary, with the same quantity of labour a less quantity of game or a greater quantity of fish was obtained, game would rise in comparison with fish (RICARDO ([1817] 1996), p. 28).

The relationship between labour costs and natural prices is proportional in this case. RICARDO notes that 'the average strength of 1000 or 10,000 men it is said is nearly the same at all times'. So if we adopt labour as the standard then 'a commodity produced in a given time by the labour of 100 men is double the value of a commodity produced by the labour of 50 men in the same time'. So we can measure the total labour used-up to produce game and fish, before and after, to determine the cause of the variationin relative prices. RICARDO therefore claims that 'the quantity of labour bestowed on a commodity ... is under many circumstances an invariable standard' (RICARDO ([1817] 1996), p. 19).

But in general there is not a proportionate relationship between the 'labour bestowed on a commodity' and its price. The problem of an invariable measure of value arises because the classical labour theory of value cannot fully account for natural prices. RICARDO (2005) clearly identifies the key problem: price depends on the distribution of income but the labour-embodied in commodities does not; therefore *the relative values of commodities vary independently of their absolute values*.

This is very perplexing, since it is equivalent to discovering that the relative size of two plots of land can change even though their absolute sizes, measured in metres, remain unaltered. Such a discovery would imply the metre is not an invariable measure of size, or that one's theory of size is flawed.

RICARDO understands the necessity for an invariable standard in his theoretical framework yet simultaneously understands the conditions that prevent this necessity from being met. Faced with a contradiction he is forced to draw the negative conclusion that there cannot be an invariable measure of value.

> It must then be confessed that there is no such thing in nature as a perfect measure of value, and that all that is left to the Political Economist is to admit that the great cause of the variation of commodities is the greater or less quantity of labour that may be necessary to produce them, but that there is also another though much less powerful cause of their variation which arises from the different proportions in which finished commodities may be distributed between master and workman in consequence of either the amended or deteriorated condition of the labourer, or of the greater difficulty or facility of producing the necessaries essential to his subsistence (RICARDO (2005), p. 404-405).

Yet RICARDO does not abandon his labour theory of value. He asserts to know of 'no other criterion of a thing being dear or cheap but by the sacrifices of labour made to obtainit' (RICARDO (2005), p. 397). The 'less powerful cause' of the variation of relative prices, that is income distribution, is an additional factor that interferes with the practical task of measuring how changes in labour productivity affect natural prices (COLLIOT-THÉLÈNE 1979). RICARDO therefore retreats to an explanation of natural prices that is necessarily approximate. He proposes to rank all possible 'imperfect' standards of value according to the extent they *minimise* the effect of changes in the distribution of in come (RICARDO 2005, SRAFFA 2005).

RICARDO observes that 'a commodity produced by labour alone in one day is totally unaffected by a variation in profits, and a commodity produced in oneyearis less affected by a variation in profits than a commodity produced in two' (RICARDO (2005), p. 404). The most useful standard is then an 'average commodity' that falls in--between those commodities produced by labour alone and those commodities produced by the maximum advances of money-capital. RICARDO suggests that we adopt acommodity 'produced by labour employed for a year' (RICARDO (2005), p. 405). At least a conventional standard, albeit imperfect, has the advantage 'that we may at least understand each other when we are talking of the rise or fall in the value of things' (RICARDO (2005), p. 371).[20] But despite

[20] Sraffa claims that Ricardo 'was not interested for its own sake in the problem of why two commodities produced by the same quantities of labour are not of the same exchangeable value' (Sraffa (2005) ,p. xlix). On this reading Ricardo's search for an invariable standard is primarily motivated by the requirementto explain the effect on profits of a change in wages (Srffa (2005), p. xlvii). Sraffa proposes to separate the search for a standard invariant to changes in technology from the search for a standard invariant to changes in the

RICARDO's efforts he bequeathed an unstable theoretical system that eventually led to the rejection of his theory of value (RUBIN (1979), Ch. 33).

10. **An invariable measure of value**

RICARDO's problem of an invariable measure derives from an accounting error in the measurement of absolute value. Money-capital has a price, the rate of profit, but also an associated labour cost, the direct and indirect labour used-up to produce capitalist consumption goods per unit of money-capital advanced. Prices of production count the price of money-capital as a monetary cost of production. But standard labour-values do not count the labour-cost of money-capital as a real cost of production. In consequence, relative values can vary independently of absolute values. The same labour-cost accounting error that causes the transformation problem is also the cause of the problem of an invariable measure of value.

distribution of income. Sraffa (1960) defines a composite collection of commodities for a given technique, called the 'standard commodity', which has the property that its price is invariant to changes in the distribution of income (Bellino 2004). The standard commodity therefore meets Ricardo's requirement for an 'average commodity' that minimises the non-proportionate effects of changes in the distribution of income. The inverse relation between the money wage and the rate of profit is linear once wages and profits are measured in terms of amounts of the standard commodity they buy (Pasinetti (1977), p. 112-120). Nevertheless, as Colliot-Thélène (1979) explains, 'Ricardow as equally concerned to establish *a relation of cause and effect* between the variations in productivity in the different branches of production and variations in prices' (see also Bidard (2004), Ch. 7). Sraffa's standard commodity does not meet Ricardo's requirement for an invariable measure that controls for the interference of income distribution in order to trace how changes in the 'difficulty of production' (absolute value) affect prices (relative or exchangeable value). Also, Sinha (2000) surveys the attempts to transpose Marx's concept of exploitation into the Sraffian framework and concludes that the standard commodity cannot solve Marx's transformation problem.

As competitive prices are proportional to nonstandard labour-values there is nota 'less powerful cause' of the variation of relative values in addition to labour costs. Once 'difficulty of production' is correctly measured *the relative values of commodities do not vary independently of their absolute values*. A search for an invariable measure of value, which controls for the effect of changes in the distribution of income, is not required. Under the conditions of the problem the 'quantity of labour bestowed on a commodity' is under all circumstances an invariable standard.

Consider that we observe prices of production $\mathbf{p}_1$; then, at a later date, we observeprices ofproduction $\mathbf{p}_2$. In the intervening period many events may have occurred, including a change in the price level, technique, or income distribution. Irrespective of these events wedo 'have the means of ascertaining' which commodities 'fell and which roseinreal value' (RICARDO ([1817] 1996), p. 38) because prices of production directly represent the 'difficulty of production' of commodities. By equation (26) the nonstandard labour-values in each period are $\tilde{\mathbf{v}}_1 = \mathbf{p}_1/w_1$ and $\tilde{\mathbf{v}}_2 = \mathbf{p}_2/w_2$, where w_1 and w_2 are the prevailing money wage rates. Let $\mathbf{x}[i]$ denote the ith element of vector $\mathbf{x}$. Then the 'difficulty of production' of commodity i has decreased if $\tilde{\mathbf{v}}_1[i] > \tilde{\mathbf{v}}_2[i]$, increased if $\tilde{\mathbf{v}}_1[i] < \tilde{\mathbf{v}}_2[i]$, and is unchanged otherwise. Just as the relative size of two plots of land can change if and only if their absolute sizes change, the relative price of two commodities can change if and only if their nonstandard labour-values change. Inter-temporal comparisons of absolute value simply require controlling for inter-temporal changes in nominal wage rates; that is, relative changes in the amount of labour that money commands.[21]

[21] Adam Smith notes the equivalence of labour-embodied and labour-commanded measures of value, in states of equilibrium, in the 'early and rude state' of society prior to the accumulation of stock and ownership ofland (see Smith ([1776] 1994), p. 54). The equivalence is also partially reproduced by Sraffa's device of the standard commodity. The quantity of labour that may be purchased with the 'standard net product' (Sraffa (1960), p. 20) is invariant to changes in the distribution of income. Sraffa remarks that it is 'surprising' that this measure 'should be found to be equivalent to something very close to the standard suggested by Adam Smith, namely 'labour commanded', to which Ricardo himself was so decidedly opposed'.

10.1. Remarks on the value of the net product. Nonstandard labour values depend on the real distribution of income. How can the objective 'difficulty of production' of commodities depend on how the net product is divided between workers and capitalists?

Consider a net product $\mathbf{n} = \mathbf{w}+\mathbf{c}$ that gets distributed in the form of the real wage $\mathbf{w}$ and capitalist consumption goods $\mathbf{c}$. The total 'difficulty of production' of the net product is clearly independent of the distribution of income in two senses: first, the total amount of direct labour used-up to produce the net product, $\mathbf{ln}^T$, cannot change depending on how it is divided; second, the total amount of labour used-up over multiple periods to produce the net product also cannot change depending on how it is subsequently divided. In sum, both *direct* and *historical* labour costs are necessarily independent of the distribution of income.

Such facts suggest the metaphor of net product as a homogeneous 'cake'. Cutting a cake into slices does not change its size. A distributional conflict between workers and capitalists over shares of the net product or 'surplus' therefore follows. For example, RICARDO considers the absolute value of the net product as a *given* magnitude that *breaks down* into wages and profits (RUBIN 1979). If workers receive a larger share of the value of the net product then of necessity capitalists receive a smaller share. For example, MARX writes 'the distribution or appropriation of value is certainly not the source of the value that is appropriated. If this appropriation did not take place, and the workman received the whole product ofhislabour as wage, the value ofthe commodities produced wouldbejustthe same as before, although it would not be shared with the landowner or capitalist' (MARX (2000), p. 94).

But labour-values are replacement costs; they are not direct or historical costs. The 'cake' metaphor misleads because the replacement cost of the net product does change with its division. To see this recall that labour-values measure the total direct and indirect labour that would be required to replace commodities given the 'present-day production conditions' of the economy. In the nonstandard approach the production conditions include the current consumption levels of of both workers and capitalists, whether that consumption be subsistence, conventional or induced. Assume that the technique **A** and direct labour coefficients **l** are constant. Consider the net product **n** is divided such that $\mathbf{n} = \mathbf{w}_1 + \mathbf{c}_1$; then later the division changes such that $\mathbf{n} = \mathbf{w}_2 + \mathbf{c}_2$, where $\mathbf{w}_1 \neq \mathbf{w}_2$ and $\mathbf{c}_1 \neq \mathbf{c}_2$. A change in the division of the net product is a change in production conditions. In particular, the objective cost of maintaining the capitalist class has changed. Since the capitalist class consumes a different basket of goods during the period of replacement a different amount of total labour is now required to replace goods 'from scratch' *independent of any corresponding changes in the real wage* due to the assumption of a constant net product.[22] In consequence, the labour--value of the net product changes with its division. Although the net product considered as a heterogeneous collection of *use-values* cannot materially change depending on how itis

[22] See section 6 on the irreducibility of the standard unit of measure.

distributed when considered as a homogeneous quantity of *exchange-value* it can.[23]

The existence of a distributional conflict does not require that the labour-value of the net product be invariant to changes in the real distribution of income (the 'cake' metaphor).[24] Price equation (2) implies that $\mathbf{p} = \mathbf{l}\,[\mathbf{I} - \mathbf{A}(1+r)]^{-1}w(1+r)$. By the proportionality of prices and nonstandard labour-values (equation (26)) and the equality of the price and labour-value rate of profit (equation (27)) it follows that $\tilde{\mathbf{v}} = \mathbf{l}[\mathbf{I} - \mathbf{A}(1+r)]^{-1}(1+r)$. Substitute this expression into the wage value equality (equation (17)) toget

$$L = \mathbf{l}[\mathbf{I} - \mathbf{A}(1+r)]^{-1}(1+r)\mathbf{w}^{\mathrm{T}}.$$

Notethat matrix $[\mathbf{I} - \mathbf{A}(1+r)]^{-1}(1+r)$ is an increasing function of r. Assuming the technique **A**, direct labour coeficients **l** and the total available labour force L is fixed then it follows

[23] For Ricardo capitalism is a 'natural' order with economic laws both immutable and ultimately reducible to physical laws, such as 'the biological law of population and the physico-chemical law of the declining fertility of the soil' (Rubin (1979), p. 243). His search for an 'absolute' measure of 'difficulty of production', independent of the institutional structure of the capitalist economy, is consistent with this outlook. Butas Roncaglia (2005) explains, 'No society exists devoid of social institutions, and the idea of an absolute value, grounded on exclusively natural foundations, is therefore a chimera'. Quantities of money-capital supplied, whether in the form of gold, paper money or data, are as much part of 'physical' reality as any other goods and services. Nonetheless the existence of a capitalist class that funds the period of production and receives profit income is a social not a natural property of an economic system. From a 'natural' perspective it is therefore difficult to countenance that the advance of money-capital incurs an indirect labour-cost that is partly constitutive of the objective 'difficulty of production' of commodities. Ricardo's theoretical difficulties may therefore ultimately derive from his attempt to explain irreducibly social properties exclusively in terms of natural properties.

[24] For example, Sraffa (1960) constructs the standard commodity to reveal a trade-off between the rate of profit and the share of wages without recourse to a theory of value (Sraffa1960).

that an increase (resp. decrease) in the rate of profit, r, entails a decrease (resp. increase) in the real wage $\mathbf{w}$. Alternatively, let $\sigma = \tilde{\mathbf{v}}\mathbf{w}^T/\tilde{\mathbf{v}}\mathbf{n}^T$ be the labour-value share of workers in the net product; then

$$\sigma = \frac{1}{\tilde{e}+1},$$

where $\tilde{e}$ is the rate of exploitation.[25] An increase (resp. decrease) in the rate of exploitation reduces (resp. increases) workers share in the net product. If the rate of exploitation is zero workers receive the full value of the net product. Since prices are proportional to nonstandard labour-values s also represents the monetary share.

10.2. Remarks on the multiplicity of measures of absolute value. On purely logical grounds the 'difficulty of production' of commodities can in principle be measured in terms of physical units of any commodity, such as units of corn, iron etc; that is, there are as many kinds of standard units, or real-cost bases, as there are kinds of commodities.

BRÓDY (1970) introduces methods of reduction for real-cost analogues of standard labour-values. WRIGHT (2007) introduces methods of reduction for real-cost analogues of nonstandard labour-values. It turns out that prices of production are not only proportional to nonstandard labour-values, but also nonstandard corn-values, iron-values etc. In consequence, any real-cost basis can function as an invariable measure of value. RICARDO to some extent perceived that a single solution to the problem of an invariable measure immediately implies a plurality of solutions. He writes, in response to Malthus' disquisition on the subject,

[25] $\sigma = \tilde{\mathbf{v}}\mathbf{w}^T/(\tilde{\mathbf{v}}\mathbf{c}^T+\tilde{\mathbf{v}}\mathbf{w}^T = 1/((\tilde{\mathbf{v}}\mathbf{c}^T/\tilde{\mathbf{v}}\mathbf{w}^T) +1) = 1/(e+1)$.

> Labour says Mr. Malthus never varies in itself, a day's labour is always worth a day's labour, therefore labour is invariable and a goodmeasure of value. In this way I might prove that no commodity ever varied and therefore that any one was equally applicable as a measure of value, as for example gold never varies in itself and therefore is an invariable measure of value – cloth never varies in itself and therefore is an invariable measure of value ... (RICARDO (2005), p. 392)[26]

A state of self-replacing equilibrium exhibits a great deal of symmetry precisely because it represents only mutual and simultaneous consistency rather than causal change. In this special case there are a surfeit of real-cost measures to choose from; therefore the theory of valueis under-determined. In consequence, progress in the theory of value necessarily requires an analysis of causal laws rather than logical consistency. Linear production theory is therefore an insufficient basis for the further development of the objective theory of economic value.

RICARDO states that if we were 'in possession of the knowledge of the law which regulates the exchange able-value of commodities, we should be only one step from the discovery of a measure of absolute value' (RICARDO (1951), p. 377). In MARX's theory the 'law of value', an unintended consequence of distributed production and market exchange (RUBIN 1973, WRIGHT 2008), is a social mechanism that measures the 'difficulty of production' of commodities independent of the subjectivity of economic actors.[27] MARX's

[26] Ricardo immediately dismisses this possibility since prices are not proportional to labour-values due to the profits on money-capital advanced (Ricardo (2005), p. 393).

[27] See Wright (2008) for a dynamic analysis of the law of value in the context of simple commodity production. The out-of-equilibrium mismatches between labour-embodied and labour-commanded measures of value is the mechanism by which labour resources are reallocated to meet social demand.

comment that 'the problem of an "invariable measure of value" was simply a spurious name for the quest for the concept, the nature, of value itself' (MARX (2000), Ch. 20, p. 134) is therefore apposite. A search for an explanatory theory of the economic laws that cause money to represent labour-value should replace RICARDO's search for an analytical definition of an invariable measure. PILLING (1986) explains that MARX's 'critique of political economy was *not* one which involved him finding a "constant" in terms of which everything could be quantified but of establishing the laws of mediation through which the "essence" of phenomena manifested itself as "appearance"'. Since MARX believed he had demonstrated that money represents abstract labour and also resolved the contradiction between the law of value and uniform profits he had no theoretical difficulty applying the labour theory of value to the analysis and critique of capitalist production. MARX therefore had little interest in RICARDO's problem: 'if, for example, the value of money changes, it changes to an equal degree in relation to all other commodities. Their relative values are therefore expressed in it just as correctly as if the value of money had remained unchanged. The problem of finding an invariable measure of value is there by eliminated' (MARX (2000), Ch. 20, p. 133).

11. **Conclusion**

A labour-cost accounting error is the root cause of the transformation problem and the problem of an invariable measure of value. Nonstandard labour-values, a generalisation of standard labour-values, avoid the error. The standard objections to thelogical possibility of a labour theory of valuedo not apply. In consequence, nonstandard labour-values can

form the basis for the further development of the labour theory of value.

Appendix

Proof of $r = \mathbf{p}\bar{\mathbf{c}}^{\mathrm{T}}$. First, note that the total money--capital advanced is $M = \mathbf{q}\mathbf{m}^{\mathrm{T}}$. Then

$$M = \mathbf{p}\mathbf{A}^{+}\mathbf{q}^{\mathrm{T}}.$$

Expanding,

$$M = \mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}} + \mathbf{p}\bar{\mathbf{w}}^{\mathrm{T}}\mathbf{l}\mathbf{q}^{\mathrm{T}}.$$

Then

$$M = \mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}} + \frac{1}{\mathbf{q}\mathbf{l}^{\mathrm{T}}}\mathbf{p}\mathbf{w}^{\mathrm{T}}\mathbf{l}\mathbf{q}^{\mathrm{T}} \tag{31}$$

$$= \mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}} + \mathbf{p}\mathbf{w}^{\mathrm{T}}, \tag{32}$$

as $\bar{\mathbf{w}} = \mathbf{w}/L$.

Second, multiply both sides of quantity equation (1) by prices $\mathbf{p}$,

$$\mathbf{p}\mathbf{c}^{\mathrm{T}} = \mathbf{p}\mathbf{q}^{\mathrm{T}} - \mathbf{p}\mathbf{A}\mathbf{q}^{\mathrm{T}} - \mathbf{p}\mathbf{w}^{\mathrm{T}}, \tag{33}$$

and substitute (32) into (33) to get

$$\mathbf{p}\mathbf{c}^{\mathrm{T}} = \mathbf{p}\mathbf{q}^{\mathrm{T}} - \mathbf{m}\mathbf{q}^{\mathrm{T}}.$$

Then

$$\begin{aligned} \mathbf{p}\mathbf{c}^{\mathrm{T}} &= \mathbf{m}\mathbf{q}^{\mathrm{T}}(1+r) - \mathbf{m}\mathbf{q}^{\mathrm{T}} \\ &= \mathbf{m}\mathbf{q}^{\mathrm{T}}r, \end{aligned}$$

as $\mathbf{p} = \mathbf{m}(1+r)$. But $M = \mathbf{q}\mathbf{m}^{\mathrm{T}}$; therefore

$$\mathbf{p}\mathbf{c}^{\mathrm{T}} = Mr.$$

As $\bar{\mathbf{c}} = \mathbf{c}/M$,

$$\mathbf{p}\bar{\mathbf{c}}^{\mathrm{T}} = r,$$

as required.

Equivalence of (13) and (18). Multiply equation (13), $\tilde{\mathbf{v}} = \tilde{\mathbf{v}}\mathbf{A} + \tilde{\mathbf{v}}\bar{\mathbf{c}}^{\mathrm{T}}\mathbf{m} + \mathbf{l}$, by $\mathbf{q}^{\mathrm{T}}$ to get

$$\begin{aligned}\tilde{\mathbf{v}}\mathbf{q} &= \tilde{\mathbf{v}}\mathbf{A}\mathbf{q}^{\mathrm{T}} + \tilde{\mathbf{v}}\bar{\mathbf{c}}^{\mathrm{T}}\mathbf{m}\mathbf{q}^{\mathrm{T}} + \mathbf{l}\mathbf{q}^{\mathrm{T}} \\ &= \tilde{\mathbf{v}}\mathbf{A}\mathbf{q}^{\mathrm{T}} + \tilde{\mathbf{v}}\mathbf{c}^{\mathrm{T}} + \mathbf{l}\mathbf{q}^{\mathrm{T}}\end{aligned} \tag{34}$$

The value rate of profit is

$$r_v = \frac{\tilde{\mathbf{v}}\mathbf{c}^{\mathrm{T}}}{\tilde{\mathbf{v}}\mathbf{A}\mathbf{q}^{\mathrm{T}} + \tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}}}.$$

Hence $\tilde{\mathbf{v}}\mathbf{c}^{\mathrm{T}} = (\tilde{\mathbf{v}}\mathbf{A}\mathbf{q}^{\mathrm{T}} + \tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}})r_v$. Substitutethis expression into (34) to get

$$\tilde{\mathbf{v}}\mathbf{q}^{\mathrm{T}} = \tilde{\mathbf{v}}\mathbf{A}\mathbf{q}^{\mathrm{T}} + (\tilde{\mathbf{v}}\mathbf{A}\mathbf{q}^{\mathrm{T}} + \tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}})r_v + \mathbf{l}\mathbf{q}^{\mathrm{T}}.$$

As $\tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}} = \mathbf{l}\mathbf{q}^{\mathrm{T}}$ then

$$\tilde{\mathbf{v}}\mathbf{q}^{\mathrm{T}} = (\tilde{\mathbf{v}}\mathbf{A} + \mathbf{l})(1 + r_v)\mathbf{q}^{\mathrm{T}}.$$

As $\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}} = 1$ then

$$\begin{aligned}\tilde{\mathbf{v}}\mathbf{q}^{\mathrm{T}} &= (\tilde{\mathbf{v}}\mathbf{A} + \tilde{\mathbf{v}}\bar{\mathbf{w}}^{\mathrm{T}}\mathbf{l})(1 + r_v)\mathbf{q}^{\mathrm{T}} \\ &= \tilde{\mathbf{v}}\mathbf{A}^{+}(1 + r_v)\mathbf{q}^{\mathrm{T}}\end{aligned}$$

Hence

$$\tilde{\mathbf{v}}(\mathrm{I} - \mathbf{A}^{+}(1+r_v))\mathbf{q}^{\mathrm{T}} = 0,$$

which is a dot product equation $\mathbf{x} \cdot \mathbf{q} = 0$, where $\mathbf{x} = \tilde{\mathbf{v}}(\mathrm{I} - \mathbf{A}^{+}(1+r_v))$. Either (i) $\mathbf{x}$ is orthogonal to $\mathbf{q}$ or (ii) $\mathbf{x}$ or $\mathbf{q}$

is $\mathbf{0}$. Consider case (i): $\mathbf{q}$ is a non-negative vector hence any orthogonal vector $\mathbf{y} = [y_i]$ must have at least one $y_i < 0$. $\mathbf{x}$ is a non-negative vector. Hence $\mathbf{x}$ is not orthogonal to $\mathbf{q}$. Consider case (ii): $\mathbf{q} \neq \mathbf{0}$ hence $\mathbf{x} = \mathbf{0}$; that is

$$\tilde{\mathbf{v}} = \tilde{\mathbf{v}}\mathbf{A}^{+}(1 + r_v), \tag{35}$$

as required. Eigenvector equation (35) and constraint $\tilde{\mathbf{v}}\bar{\mathbf{w}}^{\mathrm{T}} = 1$ determine $\tilde{\mathbf{v}}$.

Proportionate growth and nonstandard labour--values. Consider an economy that produces a net product $\mathbf{n}(t)$ at time t. A part of the net product $\mathbf{w}(t)$ is devoted to worker consumption, a part $\mathbf{c}(t)$ is devoted to capitalist consumption, and a part $\mathbf{i}(t)$ is not consumed but invested as additional means of production; that is

$$\mathbf{q}(t) = \mathbf{q}(t)\mathbf{A}^{\mathrm{T}} + \mathbf{w}(t) + \mathbf{c}(t) + \mathbf{i}(t). \tag{36}$$

Proportionate growth implies that new investment is proportional to the existing means of production, $\mathbf{i}(t) = g\mathrm{q}(t)\mathbf{A}^{\mathrm{T}}$, where g is the growth rate of the economy. Assume constant returns to scale.

Worker and capitalist consumption, or final demand, grows at rate g proportionate to the initial real distribution of income; that is, $\mathbf{w}(t) = \mathbf{w}(0)(1+g)^t$ and $\mathbf{c}(t) = \mathbf{c}(0)(1+g)^t$.

Substitute the growth assumptions into (36) to get

$$\mathbf{q}(t) = \mathbf{q}(t)\mathbf{A}^{\mathrm{T}}(1 + g) + [\mathbf{w}(0) + \mathbf{c}(0)](1 + g)^t. \tag{37}$$

Solve (37) to yield an expression for the evolution of the gross product over time,

$$\mathbf{q}(t) = (1 + g)^t[\mathbf{w}(0) + \mathbf{c}(0)](\mathbf{I} - (1 + g)\mathbf{A}^{\mathrm{T}})^{-1}. \tag{38}$$

Proportionate growth in investment and final demand implies proportionate growth of the gross product. From (37), $\mathbf{q}(0) = \mathbf{q}(0)\mathbf{A}^{T}(1+g) + \mathbf{w}(0) + \mathbf{c}(0)$; hence $\mathbf{w}(0) + \mathbf{c}(0) = \mathbf{q}(0)(\mathbf{I} - (1+g)\mathbf{A}^{T})^{-1}$. Substitute for $\mathbf{w}(0) + \mathbf{c}(0)$ in (38) to give,

(39) $$\mathbf{q}(t) = \mathbf{q}(0)(1+g)^{t},$$

as required.

The increase of the physical scale of the economy implies a corresponding growth of the total workforce, or length of the working day,

$$L(t) = \mathbf{l}\mathbf{q}^{T}(t) = \mathbf{l}\mathbf{q}^{T}(0)(1+g)^{t}.$$

The economyisgrowingbutper capita consumption(the wage rate) is constant over time,

$$\bar{\mathbf{w}}(t) = \frac{\mathbf{w}(t)}{L(t)} = \frac{\mathbf{w}(0)(1+g)^{t}}{\mathbf{l}\mathbf{q}(0)(1+g)^{t}} = \bar{\mathbf{w}}.$$

Hence the technique augmented by workers consumption, $\mathbf{A}^{+} = \mathbf{A} + \bar{\mathbf{w}}^{T}(t)\mathbf{l} = \mathrm{A} + \bar{\mathbf{w}}^{T}\mathbf{l}$, is also constant over time.

Prices and the rate of profit are invariant under proportionate growth (Pasinetti (1977), Ch. 7). Cost prices $\mathbf{m} = (1/(1+r))\mathbf{p}$. The growth in the total money-capital advanced in each period is

$$M(t) = \mathbf{m}\mathbf{q}^{T}(t) = \mathbf{m}\mathbf{q}^{T}(0)(1+g)^{t}.$$

Hence capitalist consumption per unit of money-capital advanced,

$$\bar{\mathbf{c}}(t) = \frac{\mathbf{c}(t)}{M(t)} = \frac{\mathbf{c}(0)(1+g)^{t}}{\mathbf{m}\mathbf{q}(0)(1+g)^{t}} = \bar{\mathbf{c}},$$

and new capital goods produced per unit of money-capital advanced,

$$\bar{\mathbf{i}}(t) = \frac{\mathbf{i}(t)}{M(t)} = \frac{g\mathbf{q}(0)\mathbf{A}^{T}(1+g)^{t}}{\mathbf{m}\mathbf{q}(0)(1+g)^{t}} = \bar{\mathbf{i}},$$

are both constant over time. The price of money-capital, or rate of profit, equals the rate of capitalist expenditure on consumption and investment goods, $r = \mathbf{p}[\bar{\mathbf{c}}^{T} + \bar{\mathbf{i}}^{T}]$.

For example, consider that the 2-commodity economy introduced in section 4 starts growing at rate $g = 0.1$ (that is, 10% growth per 'year'). From (38) the gross product is $\mathbf{q}(t) = [15.73 \quad 18.539] \times 1.1^{t}$, $L(t) = 175.843 \times 1.1^{t}$, and the real wage rate is $\bar{\mathbf{w}} = [0.011 \quad 0]$. $\mathbf{A}^{+} = \mathbf{A} + \bar{\mathbf{w}}\,\mathbf{l}^{T} = \mathbf{A} + \begin{bmatrix} 0.114 & 0.011 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0.614 & 0.211 \\ 0.6 & 0.4 \end{bmatrix}$. The eigenvalue equation $\mathbf{p}\mathbf{A}^{+} = \lambda\mathbf{p}$ yields the characteristic equation $\lambda^{2} - 1.014\lambda + 0.119 = 0$. The dominant root is $\lambda_{*} = 0.879$; hence the rate of profit $r = 0.138$ or 13.8%. Solving the eigenvector equation $\mathbf{p}(\mathbf{A}^{+} - \lambda_{*}\mathbf{I}) = \mathbf{0}$ yields $\mathbf{p} = p_{1}[1 \quad 0.442]$ where p_{1} is the *numéraire*. Thus $\mathbf{m} = 1/(1+r)\mathbf{p} = p_{1}[0.879 \quad 0.388]$, $M = \mathbf{q}(t)\mathbf{m}^{T} = 21.015 \times 1.1^{t}p_{1}$, the capitalist consumption rate $\bar{\mathbf{c}} = (1/p_{1})[0.103 \quad 0.08]$, and the real investment rate $\bar{\mathbf{i}} = (1/p_{1})[0.055 \quad 0.08]$. The price of money-capital equals the price of capitalist consumption and investment, $r = \mathbf{p}[\bar{\mathbf{c}}^{T}(t) + \bar{\mathbf{i}}^{T}(t)] = 0.138$ money units per unit of money-capital, as expected.

Nonstandard labour-values are constant during proportionate growth. Define the technique augmented by capitalist consumption and real investment as $\grave{\mathbf{A}} = \mathbf{A} + \bar{\mathbf{c}}^{T}\mathbf{m} + \bar{\mathbf{i}}^{T}\mathbf{m} = [\grave{a}_{i,j}]$. Nonstandard labour-values are

(40) $$\begin{aligned} \tilde{\mathbf{v}} &= \tilde{\mathbf{v}}\grave{\mathbf{A}} + \mathbf{l} \\ &= \tilde{\mathbf{v}}\mathbf{A} + \tilde{\mathbf{v}}(\bar{\mathbf{c}}^{T} + \bar{\mathbf{i}}^{T})\mathbf{m} + \mathbf{l}, \end{aligned}$$

where the scalar $\tilde{\mathbf{v}}(\bar{\mathbf{c}}^{\mathrm{T}} + \bar{\mathbf{i}}^{\mathrm{T}})$ is the labour-value of money--capital, which measures how much direct and indirect labour is used-up per unit of money-capital advanced. In conditions of proportionate growth, therefore, nonstandard labour-values are equivalent to the assumption that both capitalist consumption goods and investment goods are produced during the hypothetical period of replacement.

Nonstandard labour-values can be equivalently defined as

$$\begin{aligned} \tilde{\mathbf{v}} &= \tilde{\mathbf{v}}\mathbf{A}^{+}(1+r_v) \\ \tilde{\mathbf{v}}\bar{\mathbf{w}}^{\mathrm{T}} &= 1, \end{aligned} \tag{41}$$

where

$$r_v = \frac{\tilde{\mathbf{v}}(\mathbf{c}^{\mathrm{T}}(t) + \mathbf{i}^{\mathrm{T}}(t))}{\tilde{\mathbf{v}}\mathbf{A}\mathbf{q}^{\mathrm{T}}(t) + \tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}}(t)} = \frac{\tilde{\mathbf{v}}\mathbf{c}^{\mathrm{T}}(0) + g\tilde{\mathbf{v}}\mathbf{A}\mathbf{q}^{\mathrm{T}}(0)}{\tilde{\mathbf{v}}\mathbf{A}\mathbf{q}^{\mathrm{T}}(0) + \tilde{\mathbf{v}}\mathbf{w}^{\mathrm{T}}(0)}$$

is the labour-value rate of profit.

For example, the technique augmented by capitalist consumption and in vestment is $\grave{\mathbf{A}} = \mathbf{A} + \bar{\mathbf{c}}^{\mathrm{T}}\mathbf{m} + \bar{\mathbf{i}}^{\mathrm{T}}\mathbf{m} = \mathbf{A} + \begin{bmatrix} 0.09 & 0.04 \\ 0.07 & 0.031 \end{bmatrix} = \begin{bmatrix} 0.59 & 0.24 \\ 0.67 & 0.431 \end{bmatrix}$.

Hence nonstandard definition (40), $\tilde{\mathbf{v}} = \tilde{\mathbf{v}}\,\grave{\mathbf{A}} + \mathbf{l}$, yields nonstandard labour-values $\tilde{\mathbf{v}} = [87.921 \ \ 38.824]$, which are proportional to prices of production, as expected.

Equivalently, nonstandard definition (41), $\tilde{\mathbf{v}}\mathbf{A}^{+} = \lambda\tilde{\mathbf{v}}$, yields characteristic equation $\lambda^2 - 1.014\lambda + 0.119 = 0$. The dominant root is $\lambda_* = 0.879$; hence the labour-value rate of profit $r_v = 0.138$ or 13.8%. Solving equation $\tilde{\mathbf{v}}(\mathbf{A}^{+} - \lambda_*\mathbf{I}) = \mathbf{0}$ yields $\mathbf{v} = \tilde{v}_1[1 \ \ 0.442]$. Solving $\mathbf{v}\bar{\mathbf{w}}^{\mathrm{T}} = 1$ gives $\tilde{v}_1 = 87.921$. Hence $\tilde{\mathbf{v}} = [\,87.921 \ \ 38.824]$.

In section 6 we observed that standard labour-values do not measure the replacement costs that obtain in self-

-replacing equilibrium, or simple reproduction. In conditions of proportionate growth, or expanded reproduction, standard labour-values measure the counter-factual replacement costs that would obtain if both capitalists abstained and investment goods were not produced during replacement. In consequence standard labour-values do not measure the actual replacement costs that obtain in expanded reproduction, a property also noted by von Weizsäcker and Samuelson (1971) in their critique of the standard labour theory of value.

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